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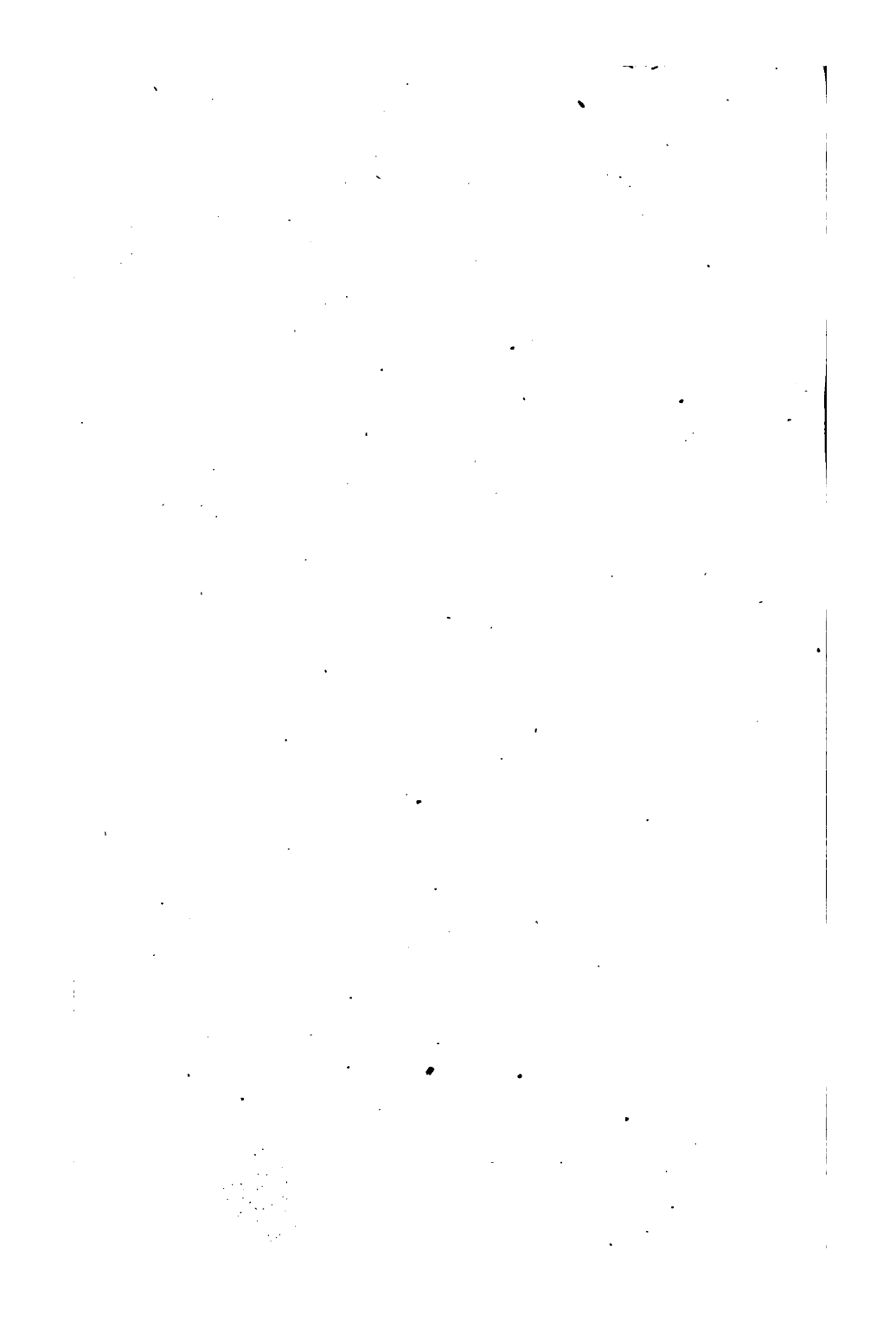
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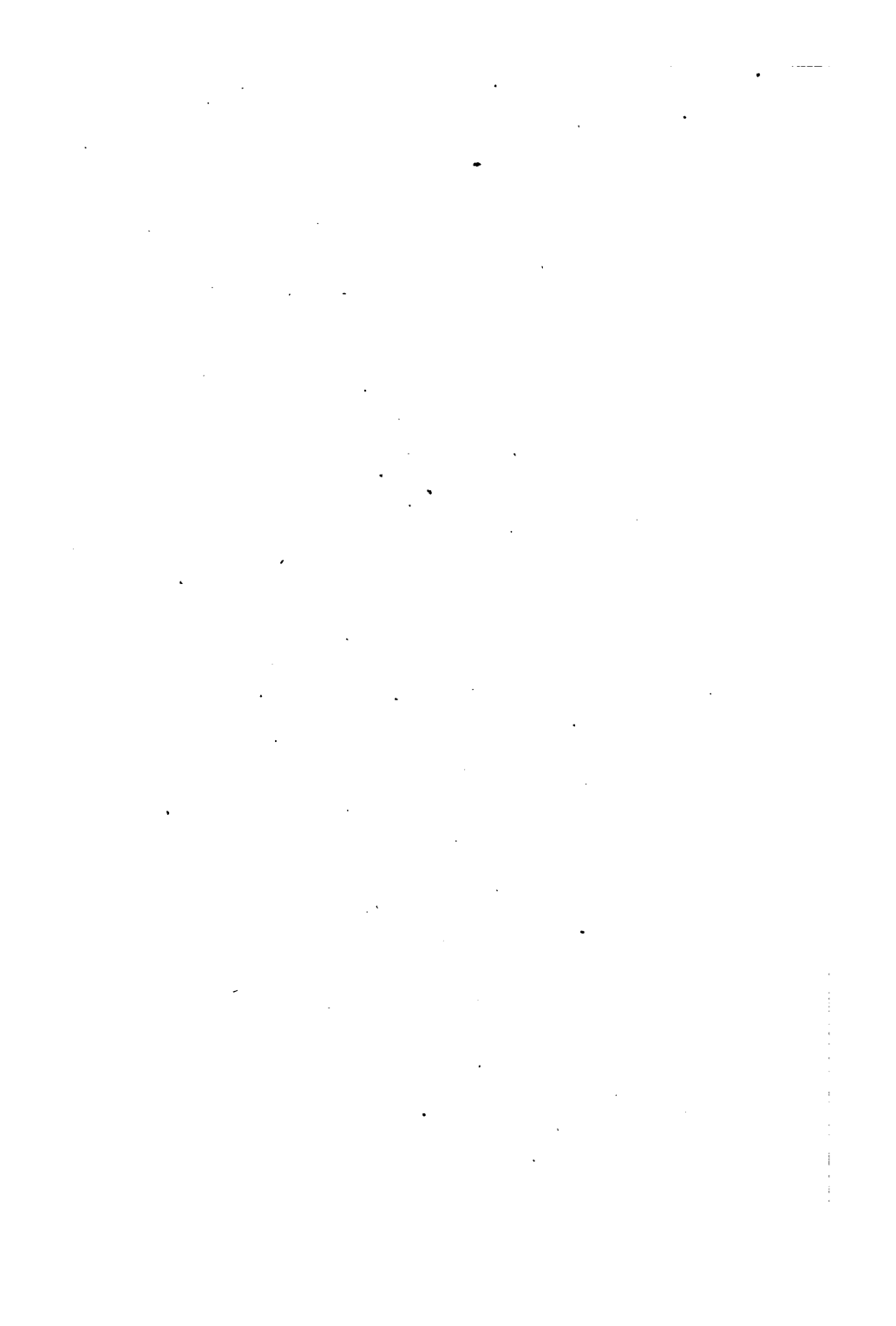
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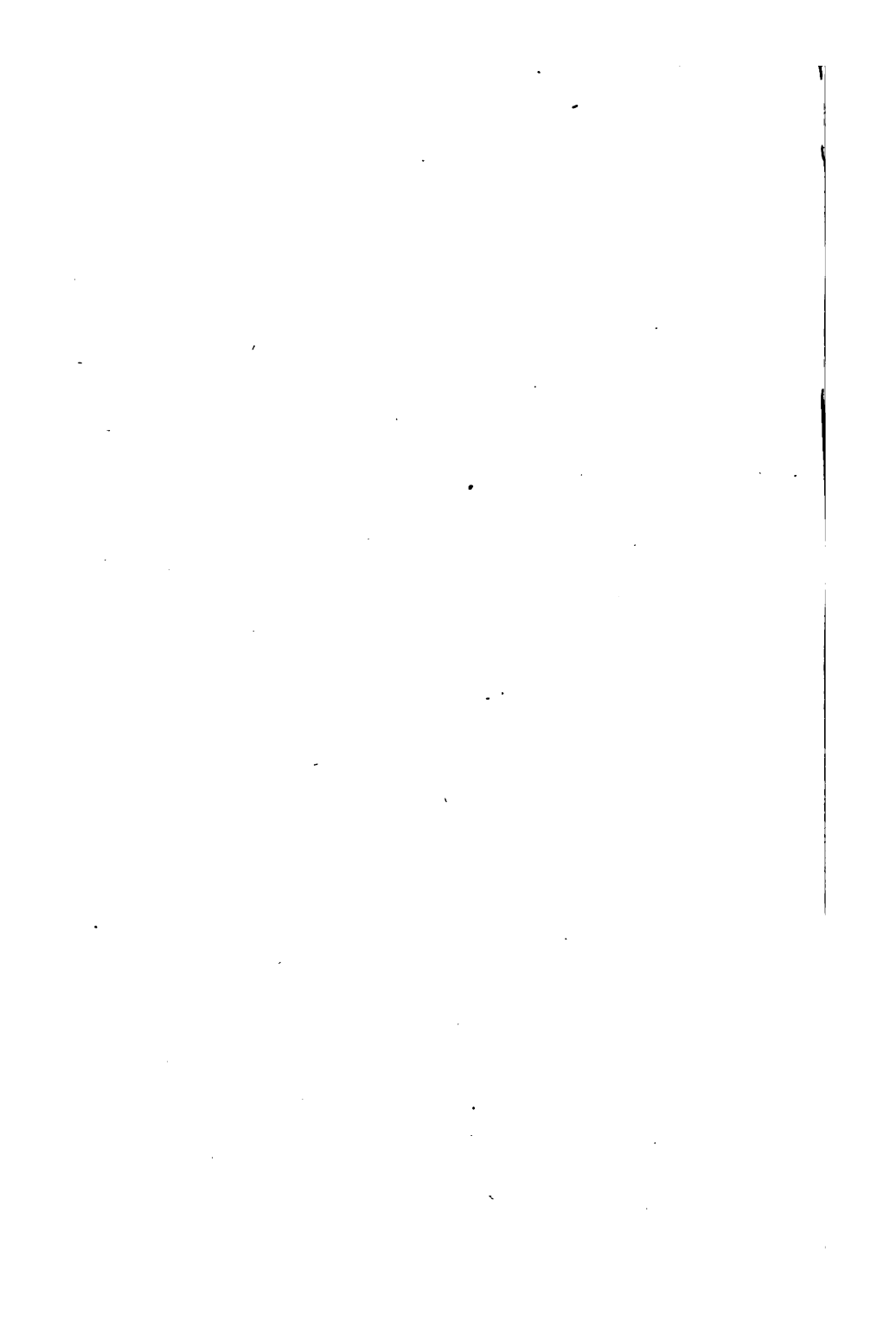
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K E Y

TO THE

EXERCISES IN ALGEBRA,

FOR THE

USE OF THE TEACHER.

BY FRANCIS J. GRUND,

Author of an "Elementary Treatise on Plane and Solid Geometry,"
"Elementary Treatise on Natural Philosophy," "Elements
of Chemistry," "Popular Lessons in Astronomy,"
translator of "Meier Hirsch's Problems," etc.

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K E Y
TO
EXERCISES IN ALGEBRA.

SECTION I.

ANSWERS TO THE EXERCISES IN ADDITION,
SUBTRACTION, MULTIPLICATION,
AND DIVISION.

A. Answers to the Exercises in Addition.

a. ADDITION OF SIMPLE QUANTITIES.

- | | |
|-------------------|-------------------|
| 1. Ans. $2a$. | 2. Ans. $14a$. |
| 3. " $6b$. | 4. " $6f$. |
| 5. " $a + b$. | 6. " $7a + 10b$. |
| 7. " 0 . | 8. " $11a$. |
| 9. " $-a$. | 10. " $a - b$. |
| 11. " $8a - 5b$. | 12. " $-7a + b$. |
| 13. " $-17a$. | 14. " $-12a$. |
| 15. " $8a + 3b$. | 16. " $6a$. |
| 17. " $-15f$. | 18. " $-3b$. |
| 19. " $-15c$. | 20. " $5d$. |
| 21. " 0 . | 22. " 0 . |

- | | |
|------------------------|---------------------------|
| 23. Ans. $8a$. | 24. Ans. b . |
| 25. " $6a$. | 26. " $5a + 4b$. |
| 27. " $10b$. | 28. " $-a - b + 4c$. |
| 29. " $4a + 2b - 4d$. | 30. " $3a - 4b - c + d$. |
| 31. " $5a + b - d$. | 32. " $15a - 19b - c$. |
| 33. " $7a - 6b - d$. | 34. " $2a - 2b$. |
| 35. " $6a + 5b$. | 36. " $19a + 18b$. |
-

b. ANSWERS TO THE ADDITION OF COMPOUND QUANTITIES.

1. Ans. $5a - 8c - 4b$.
 2. " $8a - 8b + 4c$.
 3. " $7a + 5b - 13c + 10$.
 4. " $9h + 5c - 9f - 6g$.
 5. " $12a + 11b + 11c - 17d$.
 6. " $17a - b - 2c + 5d$.
 7. " 0 .
 8. " $7a - 4x - 5y + 5z - 10$.
 9. " $a - b + 5c + 3d$.
 10. " $5a + b - 3c - 4d + g - f$.
 11. " $24a + 5c - 6d + e - f$.
 12. " $11a - 6b - d + e$.
-

B. Subtraction.

a. ANSWERS TO THE EXERCISES IN SUBTRACTION OF SIMPLE QUANTITIES.

- | | |
|---------------|----------------|
| 1. Ans. 0 . | 2. Ans. $4a$. |
| 3. " 0 . | 4. " $-2d$. |

- | | |
|---------------------|-----------------|
| 5. Ans. $7a - 5b$. | 6. Ans. $2a$. |
| 7. " $9a$. | 8. " $11a$. |
| 9. " $2a$. | 10. " $a + b$. |
| 11. " $3a + 2b$. | 12. " $-12a$. |
| 13. " $-4a$. | 14. " 0 . |
| 15. " a . | 16. " $-a$. |
| 17. " $-3a + 5b$. | 18. " -16 . |
| 19. " 9 . | 20. " 19 . |
| 21. " -5 . | 22. " -16 . |
| 23. " 0 . | 24. " 16 . |
-

b. ANSWERS TO THE EXERCISES IN SUBTRACTION OF
COMPOUND QUANTITIES.

1. Ans. $a - 2b$.
2. " $a - 12b$.
3. " $3d + 10f$.
4. " $a + 5b + 9$.
5. " $5a + 4b + 7d$.
6. " $23b$.
7. " $-f + 8m - 6x - 3d$.
8. " $-7a + 2e - 5h + c$.
9. " $-a - 9b + 10c + d - 3k$.
10. " $-6h + 6k + 7 + 2l$.
11. " $3h - 9l + 6k + 7$.
12. " $3h - 2a + 5k$.
13. " $-5a + b + c - e - 5f + 7$.
14. " $2a$.
15. " $7b + 5a + 136c - 2$.
16. " $9a - 26b - 3d$.

17. Ans. $3c + 12 + 7c$.
 18. " $8a - 6c - 7d + 5e + d - e - f - 6g + 7k - 24$.
 19. " $27a - 14b$.
 20. " $20a - 5c - 3f + 5x$.
 21. " $-15a + 8b - c$.
 22. " $-4a - 13c + 8d + 2e$.
 23. " $28a + 6b - 5f - h$.
 24. " $-a + 4b$.
 25. " $7z - 10 - x$.
 26. " $7a - 5b$.
 27. " $-7a + 5b$.
 28. " $2b$.
 29. " $-a + 3b$.
 30. " $-2a + 4b$.

C. Multiplication.

a. ANSWERS TO THE EXERCISES IN MULTIPLICATION OF SIMPLE QUANTITIES.

- | | |
|--------------------------------|--------------------------|
| 1. Ans. $a b$. | 2. Ans. $b a$ or $a b$. |
| 3. " $a b$. | 4. " $b a$ or $a b$. |
| 5. " $-a b$. | 6. " $-a b$. |
| 7. " $+a b$. | 8. " $42 a b$. |
| 9. " $-42 a c$. | 10. " $-70 a b$. |
| 11. " $-7 a b$. | 12. " $66 a x$. |
| 13. " $a b c$. | 14. " $a b c$. |
| 15. " $a b c d e$. | 16. " $35 a a b c d e$. |
| 17. Ans. $-45 b b b d d x y$. | |
| 18. " $85 a c e$. | |

19. Ans. $42 a b d f g$.
 20. " $12 a b b c d e f$.
 21. " $6 a b c d d f g$.
 22. " $120 a b c d e f g h$.
 23. " $a a a a$.
 24. " $- a a a a$.
-

b. ANSWERS TO THE EXERCISES IN MULTIPLICATION OF
 COMPOUND QUANTITIES.

1. Ans. $9 a c + 6 b c$.
 2. " $6 a d + 6 b d + 10 c d$.
 3. " $21 a f + 14 b f - 35 d f + 7 e f$.
 4. " $16 a e + 20 b e - 4 d e$.
 5. " $30 a g + 15 b g - 25 f g$.
 6. " $16 b h - 24 g h + 8 c h$.
 7. " $70 a b d d - 100 b b c d$.
 8. " $30 a a b d + 90 a b b c - 96 a a b c f$.
 9. " $a c + b c + a d + b d$.
 10. " $a a + 2 a b + b b$.
 11. " $2 a c + 3 b c + 2 a d + 3 b d$.
 12. " $8 a c + 12 b c + 10 a d + 15 b d$.
 13. " $a d + b d - c d - a e - b e + c e$.
 14. " $10 a f - 15 b f - 35 c f - 12 a g + 18 b g$
 $+ 42 c g$.
 15. " $14 a f - 21 b f - 56 c f - 7 d f - 4 a g$
 $+ 6 b g + 16 c g + 2 d g - 2 a h + 3 b h$
 $+ 8 c h + d h$.
 16. " $\begin{cases} 8 a f - 12 b f - 32 c f - 4 d f + 4 g f - 2 a g \\ + 3 b g + 8 c g + d g - g g + 2 a h - 3 b h \\ - 8 c h - d h + g h \end{cases}$

17. Ans. $21\,l\,l - 83\,l\,m - 27\,l + 22\,m\,m + 99\,m.$

18. " $6\,a\,a + 35\,a\,b + 9\,a\,c + 50\,b\,b + 30\,b\,c$
 $- 10\,a\,f - 25\,b\,f - 15\,c\,f.$

19. " $9\,a\,a + 36\,a\,b + 9\,a\,c - 15\,a\,e + 20\,b\,b$
 $+ 30\,b\,c - 50\,b\,e + 3\,a\,f + 2\,b\,f + 3\,c\,f - 5\,e\,f.$

20. " $a\,a - b\,b.$

21. " $6\,a\,a + 7\,a\,b - 20\,b\,b.$

22. " $a\,a - 2\,a\,b + b\,b.$

23. " $6\,a\,a - 29\,a\,b + 35\,b\,b.$

24. " $35\,a\,a\,b\,b - 19\,a\,a\,b\,c - 28\,a\,b\,b\,c - 24\,a\,a\,c\,c$
 $+ 32\,a\,b\,c\,c.$

25. " $52\,b\,b\,c\,c\,d + 80\,b\,b\,c\,c\,e + 20\,b\,b\,c\,d\,e$
 $+ 39\,b\,b\,c\,d\,d - 30\,b\,b\,d\,d\,e - 156\,b\,c\,d\,e$
 $- 240\,b\,c\,e\,e + 120\,b\,d\,e\,e.$

26. " $15\,a\,a\,a - 14\,a\,a\,b + 24\,a\,b\,b - 7\,b\,b\,b.$

27. " $a\,a + 2\,a\,b + b\,b - c\,c.$

28. " $4\,a\,a + 14\,a\,b - 2\,a\,c + 12\,b\,b + b\,c - 20\,c\,c.$

29. " $12\,a\,a\,a\,a\,a + 20\,a\,a\,a\,a\,b\,b + 28\,a\,a\,a\,b\,b\,b$
 $- 15\,a\,a\,a\,a\,b\,b\,b - 25\,a\,a\,b\,b\,b\,b\,b$
 $- 20\,a\,b\,b\,b\,b\,b\,b + 20\,a\,b\,b\,b\,b\,b\,b$
 $+ 16\,b\,b\,b\,b\,b\,b\,b.$

30. " $51\,a\,a - 23\,a\,b + 70\,a\,c - 51\,a\,d + 2\,b\,b$
 $- 16\,b\,c + 6\,b\,d + 24\,c\,c - 36\,c\,d.$

31. " $a\,a + 2\,a\,b + b\,b - c\,c - 2\,c\,d - d\,d.$

32. " $4\,a\,a + 12\,a\,b + 9\,b\,b + 16\,a\,c - 20\,a\,d$
 $+ 24\,b\,c - 30\,b\,d + 16\,c\,c - 40\,c\,d + 25\,d\,d.$

33. " $6\,a\,f - 9\,b\,f + 3\,e\,c\,f + 14\,a\,a - 21\,a\,b$
 $+ 7\,a\,e\,c - 2\,a\,c\,c + 3\,b\,c\,c - c\,c\,c\,e.$

34. " $3\,m\,m - 11\,m\,n + 17\,m\,p\,p + 10\,n\,n$
 $- 28\,n\,p\,p - 6\,p\,p\,p\,p.$

$$35. \text{ Ans. } 3 m m m m - 4 m m n n + 2 m m p p + n n n n + p p p p.$$

$$36. \text{ " } 2 m m m m - 3 m m n n - 3 m m p p + n n n n + 2 n n p p + p p p p.$$

Division.

a. ANSWERS TO THE EXERCISES IN DIVISION OF SIMPLE QUANTITIES.

$$1. \text{ Ans. } 1.$$

$$2. \text{ Ans. } a.$$

$$3. \text{ " } a a.$$

$$4. \text{ " } a.$$

$$5. \text{ " } a a.$$

$$6. \text{ " } a.$$

$$7. \text{ " } \frac{a}{b}.$$

$$8. \text{ " } -\frac{a}{b}.$$

$$9. \text{ " } -\frac{a}{b}.$$

$$10. \text{ " } \frac{a}{b}.$$

$$11. \text{ " } -b.$$

$$12. \text{ " } +a.$$

$$13. \text{ " } 3b.$$

$$14. \text{ " } 6ab.$$

$$15. \text{ " } 5ad.$$

$$16. \text{ " } \frac{5a}{3b}.$$

$$17. \text{ " } -\frac{2a}{b}.$$

$$18. \text{ " } -\frac{3a}{b}.$$

$$19. \text{ " } \frac{7a}{2b}.$$

$$20. \text{ " } bc.$$

$$21. \text{ " } \frac{bc}{d}.$$

$$22. \text{ " } -\frac{4n}{g}.$$

$$23. \text{ " } -\frac{3he}{2}.$$

$$24. \text{ " } -\frac{3ade}{f}.$$

$$25. \text{ Ans. } -\frac{3 a a b f}{2 h k}. \quad 26. \text{ Ans. } -\frac{5 a b f f g}{k}.$$

b. ANSWERS TO THE EXERCISES IN DIVISION OF COMPOUND
QUANTITIES.

$$1. \text{ Ans. } 7 a c - 2 a d e - 3 f + 3 d.$$

$$2. \text{ " } 7 c - 2 d e - 3 f + 4 d.$$

$$3. \text{ " } \frac{3 c}{2} - d e - \frac{f}{2 a}.$$

$$4. \text{ " } \frac{6 c}{d} - \frac{2 b e}{a} - \frac{2}{3 f}.$$

$$5. \text{ " } -4 a + 3 b - \frac{2 c}{a} - \frac{1}{2 a}.$$

$$6. \text{ " } \frac{3 c}{b} - \frac{f}{b} + \frac{3 g h}{4 a b}.$$

$$7. \text{ " } \frac{3 c}{a b b} - \frac{f}{a b b} + \frac{3 g h}{4 a a b b}.$$

$$8. \text{ Ans. } a.$$

$$9. \text{ " } 2 a.$$

$$10. \text{ " } c + d.$$

$$11. \text{ " } 2 c + 2 d.$$

$$12. \text{ " } 2 a + 3 b - 5 x.$$

$$13. \text{ " } a + b.$$

$$14. \text{ " } 2 x x + 5 x - 7.$$

$$15. \text{ " } 2 a - 3 b + c.$$

$$16. \text{ " } a - b + 3 c.$$

$$17. \text{ " } a + 2 b - c.$$

$$18. \text{ " } 4 a a + 3 a - b.$$

$$19. \text{ " } 2 a + 5 b - 7 c + 3 d.$$

20. Ans. $7c - 13d + 24e - 5f$.
 21. " $2a + 5b$.
 22. " $3a - 4b$.
 23. " $12xx - 5xy - 3yy$.
 24. " $3a - 2b + 8c$.
 25. " $9a - 2b + c$.
 26. " $-5ax + 12xy$.
 27. " $a + b$.
 28. " $2a + 3b$.
 29. " $4a - 3b$.
 30. " $aa + 3ab + cc$.
 31. " $aaa + aab + abb + bbb$.
 32. " $8aaa + 12aab + 18abb + 27bbb$.
 33. " $16aaaa - 8aaa + 4aab - 2abb + bbbb$.
 34. " $3ab - 5fm - 7d$.

c. ANSWERS TO THE PARTIAL DIVISIONS, PERFORMED IN
 CASES WHERE THE DIVISOR IS NOT AN EXACT
 NUMBER OF TIMES CONTAINED IN
 THE DIVIDEND.

1. Ans. 1, and the remainder $+b$, or also $1 + \frac{b}{1-b}$.
 2. " $1 + b$, and the remainder $+bb$, or also $1 + \frac{bb}{1-b}$.
 3. " $1 + b + bb$, and the remainder bbb , or also $1 + b + bb + \frac{bbb}{1-b}$.
 4. " $1 + b + bb + bbb + bbbb$.
 5. " $1 + b + bb + bbb + bbbb + bbbbb + bbbbbb + \dots$

6. " 1, and the remainder $-b$, or also $1 - \frac{b}{1+b}$.
7. " $1 - b$, and the remainder $+b$, or also $1 - b + \frac{bb}{1+b}$.
8. " $1 - b + bb - bbb + \frac{bbbb}{1+b}$.
9. " $1 - b + bb - bbb + bbbb - bbbbbb \pm \dots$.
10. " a , and the remainder is $+aa$.
11. " $a + aa$, and the remainder $+aaa$.
12. " $a + aa + aaa + aaaa + aaaaaa$, and the remainder $aaaaa$.
13. " $a + aa + aaa + aaaa + aaaaaa + aaaaaaa + \dots$.
14. " a , and the remainder $-aa$.
15. " $a - aa + aaa$, and the remainder $-aaaa$.
16. " $a - aa + aaa - aaaa + aaaaaa \mp \dots$.
17. " 1, and the remainder $+1$.
18. " $1 + \frac{1}{a}$, and the remainder $+ \frac{1}{a}$.
19. " $1 + \frac{1}{a} + \frac{1}{aa} + \frac{1}{aaa} + \dots$.
20. " 1, and the remainder is -1 .
21. " $1 - \frac{1}{a} + \frac{1}{aa}$, and the remainder is $-\frac{1}{aa}$.
22. " $1 - \frac{1}{a} + \frac{1}{aa} - \frac{1}{aaa} \pm \dots$.
23. " 1, and the remainder is $+b$.
24. " $1 + \frac{b}{c}$, and the remainder is $\frac{bb}{c}$.

25. Ans. $1 + \frac{b}{c} + \frac{bb}{cc}$, and the remainder is $\frac{bbb}{ccc}$.
26. " $1 + \frac{b}{c} + \frac{bb}{cc} + \frac{bbb}{ccc} + \dots$
27. " 1, and the remainder is $-b$.
28. " $1 - \frac{b}{c} + \frac{bb}{cc}$, and the remainder is $-\frac{bbb}{ccc}$.
29. " $1 - \frac{b}{c} + \frac{bb}{cc} - \frac{bbb}{ccc} \pm \dots$
30. " $\frac{c}{a} + \frac{bc}{aa} + \frac{bbc}{aaa} + \frac{bbbc}{aaaa} + \&c. \dots$
31. " $\frac{c}{a}$, and the remainder is $-\frac{bc}{a}$.
32. " $\frac{c}{a} - \frac{bc}{aa}$, and the remainder is $+\frac{bbc}{aaa}$.
33. " $\frac{c}{a} - \frac{bc}{aa} + \frac{bbc}{aaa} - \frac{bbbc}{aaaa} \pm \dots$
34. " 1, and the remainder is $+2x$.
35. " $1 + 2x + 2xx$, and the remainder is $2xxx$.
36. " $1 + 2x + 2xx + 2xxx + \dots$

SECTION II.

ANSWERS TO THE SUBTRACTION, MULTIPLICATION AND DIVISION OF POWERS.

1. Ans. The 2d power.
2. " The 2d power of x .
3. " The 3d power of a .
4. " The 3d power of x .

5. Ans. The 4th power.
 6. " The 4th power of x .
 7. " The 11th power.
 8. " Of 10 such factors.
 9. " Of 15 such factors.
10. Ans. 14 such factors. 11. Ans. 7 factors, x^2 .
 12. " 3 factors, x^3 . 13. " it is equal to 1.
 14. " $\frac{1}{a^2}$. 15. " $\frac{1}{a^3}$.
 16. " $\frac{1}{a^2}$. 17. " $\frac{1}{a}$.
 18. " $\frac{1}{2^1} = \frac{1}{2}$. 19. " $\frac{1}{3^1} = \frac{1}{3}$.
 20. " $\frac{1}{2^2} = \frac{1}{4}$. 21. " $\frac{1}{2^3} = \frac{1}{8}$.
 22. " $\frac{1}{4^1} = \frac{1}{4}$. 23. " 1.

A. Addition of Powers.

a. ANSWERS TO THE EXERCISES IN ADDITION OF SIMPLE QUANTITIES.

1. Ans. $6a^3$. 2. " $15a^2$.
 3. " $x^2 + 3b^3$. 4. " $10x^2 - 11a^2$.
 5. " x^2 . 6. " $6x^2 - 9y^2$.
 7. " $-2x^2 - 3y^2$. 8. " $-5x^2 - 12x$.
 9. " $11x^2 + 17x^2y$. 10. " $12x^2y - xy^2$.
 11. " $6x^2 - 4x^2y + 8x$. 12. " $20x^4$.
 13. " 0. 14. " $13a^4$.

15. Ans. $7 a^{-1} = 7 \times \frac{1}{a} = \frac{7}{a}$.

16. " $a^{-2} = \frac{1}{a^2}$.

17. " $3.6^2 + 4.5^2$.

18. " $20.6^4 + 7.8^3$.

b. ANSWERS TO THE EXERCISES IN ADDITION OF COMPOUND QUANTITIES.

1. Ans. $4 a^2 b + 8 a b^2 - c$.

2. " $8 a^2 b c - 2 a^2 b d - 6 d$.

3. " $8 a^4 b - 3 a^2 b^2 c$.

4. " 0 .

5. " $28 a^3 b^2 c^4 - 8 b + 2^4$.

6. " $10 a^3 b c^2 - a^2 b - d + 5 a b^2$.

7. " $11 a b x^2 - 5 a + m$.

8. " $2 a b^3 c + 2 a b^2 c^2 - 4 a^4 d^5 - 4 a^2$.

9. " $11 a^3 b^2 d + a^2 b^3 d - 15 d^{-2}$.

10. " $45 a^2 b^3 - 50 a^3 b^3 c^{-1} + 4 p^3 + 10 d$.

B. Subtraction of Powers.

g. ANSWERS TO THE EXERCISES IN SUBTRACTION OF SIMPLE QUANTITIES.

1. Ans. 0 .

2. Ans. a^2 .

3. " 0 .

4. " $-4 m^4$.

5. " $6 a^5 - 4 b^5$.

6. " $2 a^4$.

7. " $10 a^3$.

8. " $a^2 + b^2$.

9. Ans. 0.

10. " a^3 .

11. " $-a^3$.

12. " $-3b^2 + 5c^2$; or $5c^2 - 3b^2$.

13. " $4a^2$.

14. " $-a^2 + b^2$; or $b^2 - a^2$.

15. " $-c^2 - d^2$.

16. " $3a^2 - a$.

b. ANSWERS IN SUBTRACTION OF COMPOUND QUANTITIES.

1. Ans. $a^2 - b^2$.

2. " $11a^3 - 6b^2 + 2c^3$.

3. " $4a^3 + 11b^2 + e^2$.

4. " $4a^3 + 3a^2 - 3ab^2 - 5cd^2e^3 + df^2$.

5. " $3a^2 - 13b^2d^3 - 2c^3d - k^2$.

6. " $10a^2b^2c^2 + d^3f - 4a^2b^2$.

7. " $18a^4b^5 + 3 + 3a - d$.

8. " $2a^2 + 2a^2b^2c^2 + 3ab^3$.

9. " $a^3b^4 - 11a^2b^5c - d^4$.

10. " $2a^4 + 8a^3b^2 + 4cd^2 + 7d + 3a^2$.

11. " $2a^{-4} + 8a^{-3}b^2 + 4c^{-1}d^2$.

12. " $2a^{-3} + a^{-4}d + d^3e - ef$.

C. *Multiplication of Powers.*

a. ANSWERS TO EXERCISES IN MULTIPLICATION OF SIMPLE QUANTITIES.

1. Ans. a^2 .

2. Ans. a^3 .

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| 3. Ans. a^4 . | 4. Ans. a^4 . |
| 5. " a^5 . | 6. " a^6 . |
| 7. " a^6 . | 8. " a^8 . |
| 9. " a^{10} . | 10. " a^{13} . |
| 11. " $3 a^4$. | 12. " $8 a^5$. |
| 13. " $20 a^8$. | 14. " $-5 a^3$. |
| 15. " $-5 a^7$. | 16. " $12 a^7$. |
| 17. " $3 a^5$. | 18. " $-20 a^9$. |
| 19. " $-a^6$. | 20. " $+a^6$. |
| 21. " $4 a^7$. | 22. " $6 a^8$. |
| 23. " $6 a^7$. | 24. " $-3 a^9$. |
| 25. " $20 a^5 b$. | 26. " $15 a^{11}$. |
| 27. " $105 a^{21}$. | 28. " $-105 a^{21}$. |
| 29. " $-48 a^{19}$. | 30. " $48 a^{17} b$. |
| 31. " $12 a^3 b^3$. | 32. " $14 a^{-12}$. |
| 33. " $42 a^{-6}$. | 34. " $-42 a^{-6}$. |
| 35. " $42 a^{-6}$. | |
| 36. " $-48 a^0 = -48 \times 1 = -48$. | |
| 37. " $12 \cdot 2^5$. | 38. Ans. $6 \cdot 3^4$. |
| 39. " $18 \cdot 4^6$. | 40. " $6 \cdot 4^{-4}$. |
| 41. " $20 \cdot 4^{-6}$. | 42. " $-60 \cdot 4^{-2}$. |
| 43. " $12 a^4 b^2$. | 44. " $50 a^5 b^9 c$. |
| 45. " $-150 a^{19} b^9 c$. | 46. " $-27 a^{13} b^{-9} c$. |
| 47. " $21 a^3 b^9 c^6$. | 48. " $21 a^3 b^{-9} c^6$. |
| 49. " $21 a^3 b^{-9} c^{-6}$. | 50. " $5 a^5 b^{19}$. |
| 51. " $60 a^6 b^{13} c$. | 52. " $60 a^6 b^6$. |
| 53. " $h^{11} l^{20} x$. | |
| 54. " $h^{-3} l^2 x = \frac{l^2 x}{h^3}$. | 55. " $h^{-3} l^2 x = \frac{l^2 x}{h^3}$. |
| 56. " $3 h^{-5} l^{-4} x^2$. | 57. " $52 a^4 b^6 c^5$. |

21. Ans. $3 k^4 - 26 k^3 l + 37 k^2 l^2 - 14 k l^3.$
22. " $6 f^5 + 7 f^4 l - 65 f^3 l^2 + 12 f^2 l^3.$
23. " $20 a^5 - 88 a^4 x + 47 a^3 x^2 - 6 a^2 x^3.$
24. " $a^8 - a^2.$
25. " $a^5 + 32 b^5.$
26. " $4 a^6 x^3 - 6 a^2 b^4 x y^2 - 6 a^4 b^2 x^2 y + 9 b^6 y^3.$
27. " $4 a^9 x^4 - 6 a^5 b^4 x^2 y^2 - 6 a^4 b^4 x^2 y$
 $+ 9 b^8 y^4$
28. " $21 a^7 - 43 a^6 b + 150 a^5 b^2 - 110 a^4 b^3$
 $+ 104 a^3 b^4 - 32 a^2 b^5.$
29. " $10 x^4 + a x^3 - 22 a^2 x^2 + 4 a^3 x + 7 a^4.$
30. " $7 a^{10} - 25 a^8 b^2 + 48 a^6 b^4 - 23 a^4 b^6$
 $+ 5 a^2 b^8.$
31. " $a^8 - 8 a^7 b + 28 a^6 b^2 - 36 a^5 b^3 + 34 a^3 b^5$
 $- 22 a^2 b^6 + 2 a b^7 + b^8$
32. " $a^4 + a^2 x^2 + x^4.$
33. " $120 a^8 b^4 - 101 a^7 b^6 + 48 a^{-2} b^8 + 21 a^6 b^8$
 $- 18 a^{-3} b^{10}$
34. " $120 a^{-8} b^4 - 101 a^{-7} b^6 + 69 a^{-6} b^8$
 $- 18 a^{-5} b^{10}.$
35. " $6 x^5 y^{12} - 4 x^5 y^{10} + 18 x^3 y^{10} + 16 x^9 y^8$
 $- 12 x^3 y^8 + 36 x^7 y^8 + 24 x^7 y^6 + 96 x^{11} y^4.$
36. " $78 a^7 b^3 + 60 a^4 b^4 - 234 a^5 b^4 - 24 a^3 b^5$
 $- 180 a^2 b^5 - 91 a^8 b^5 + 72 a b^6 - 70 a^5 b^6$
 $+ 28 a^4 b^7.$
37. " $78 a^{-8} b^3 - 174 a^{-5} b^4 - 269 a^{-2} b^5 + 22 a b^6$
 $+ 20 a^4 b^7.$
38. " $10 a^6 b^6 c^4 + 3 a^7 b^5 c^7 + 14 a^{11} b^8 c^8$
 $- 18 a^8 b^4 c^{10} + 21 a^{12} b^7 c^{11} - 30 a^{10} b^7 c^5$
 $+ 36 a^{11} b^6 c^8 - 42 a^{15} b^9 c^9.$

$$39. \text{ Ans. } 196 a^{10} c^4 - 36 a^4 b^3 c^4 + 12 a^2 b c^5 - c^6.$$

$$40. \text{ " } a^4 - 4 c^6 d^8 - 28 c^5 d^4 - 49 c^4.$$

D. Division of Powers.

a. ANSWERS TO THE EXERCISES IN DIVISION OF SIMPLE QUANTITIES.

1. Ans. 1.

2. Ans. 1.

3. " a^2 .

4. " a^2 .

5. " a .

6. " a^2 .

7. " $a b c$.

8. " $a^2 b c^2$.

9. " $-2 a$.

10. " $-2 a$.

11. " $5 a^2 b$.

12. " $-4 a b$.

13. " $-4 a b^3$.

14. " $3 a^2$.

15. " $-2 a^2 b^3$.

16. " $-\frac{3}{2} a^2 b d$.

17. " $4 a^6$.

18. " $\frac{7}{2} a^3$.

19. " $7 a^{-6}$.

20. " a^{-7} .

21. " $9 a^{-8}$.

22. " $5 a^{-8}$.

23. " $\frac{c a^{12}}{d}$.

24. " $\frac{c a^{24}}{d}$.

25. " $4 a^9 b^6$.

26. " $4 a b$.

27. " $\frac{7 a f g}{b k}$.

28. " $\frac{15 b^7}{16 a}$.

29. " $\frac{5 b^4 c}{3 a}$.

30. " $-\frac{3}{2} a^3 b^2$.

31. " $6 c^2$.

32. " $\frac{a}{4 b}$.

b. ANSWERS TO THE EXERCISES IN DIVISION OF COMPOUND QUANTITIES.

1. Ans. $3ab^2 - 5f$.
2. " $4a^3b^2 - 4f^2 + \frac{1}{2}a^3bx$.
3. " $\frac{2x^5}{a} - \frac{7x^3}{a^2} + \frac{3b^2x}{2}$.
4. " $\frac{x}{a^2} - \frac{7}{2a^2x} + \frac{3b^2}{a^2x}$.
5. " $-2ax^4 - ax + \frac{3b^2}{2a}$.
6. " $-\frac{7}{y^3} + 14 - \frac{1}{x^3}$.
7. " $\frac{3}{y^2} - 4 + \frac{1}{5x^2}$.
8. " $\frac{9}{xyz} + 27 + xyz$.
9. " $\frac{4}{xyz} + \frac{x^2y^2z^2}{2}$.
10. " $-\frac{3b^4}{2a} + \frac{5c}{2a^2b^2} - \frac{c^2}{b^2} + \frac{c^2}{ab^2}$.
11. " c^3 .
12. " $a + b$.
13. " $a^2 + 2ab + b^2$.
14. " $a^2 + 2ab + b^2$.
15. " $a^3 + 3a^2b + 3ab^2 + b^3$.
16. " $3a^3 - 5a^2b + 2ab^2$.
17. " $a^4 - 4a^3b^3 + 6a^2b^6$.
18. " $a^2 - b^2$.
19. " $a^3 - b^3$.
20. " $a^4 + b^4$.

21. Ans. $b x^6 + a^3 x^5 - 7 a^4 x^4$.
 22. " $a^2 b^3 - 5 a^5 b^4 - 2 a^8 b^5$.
 23. " $a^6 + 2 a^4 x^2 + 4 a^3 x^4 + 8 x^6$.
 24. " $a^2 - 5 a b + 6 b^2$.
 25. " $2 c^2 + 3 b c - b^2$.
 26. " $2 c^2 - 3 b c + b^2$.
 27. " $1 + a x + a^2 x^2$.
 28. " $5 a^3 b^2 c^3 - 2 a^2 b^2 c^4 - 3 a b^2 c^5 - 7 b c^5$.
 29. " $- a^{-5} x^2 + 7 a^{-1} x^3 + 8 a^3 x^4$.
 30. " $a^4 + a^3 x + a x^3 + x^4$.
 31. " $a^4 + 4 a^3 x + 12 a^2 x^2 + 16 a x^3 + 16 x^4$.
 32. " $a^7 + a^6 b + a^5 b^2 + a^4 b^3 + a^3 b^4 + a^2 b^5$
 $+ a b^6 + b^7$.

C. ANSWERS TO THE PARTIAL DIVISION, PERFORMED IN
 CASES IN WHICH THE DIVISOR IS NOT AN EXACT
 NUMBER OF TIMES CONTAINED IN
 THE DIVIDEND.

1. Ans. $a - a x + a x^2 - a x^3 + a x^4 \mp \&c. \dots$
 2. " $a + a x + a x^2 + a x^3 + a x^4 + \&c. \dots$
 3. " $1 - \frac{1}{a} + \frac{1}{a^2} - \frac{1}{a^3} + \frac{1}{a^4} \mp \&c. \dots$
 4. " $\frac{a}{x} + \frac{a}{x^2} + \frac{a}{x^3} + \frac{a}{x^4} + \&c. \dots$
 5. " $\frac{a}{b} - \frac{a d}{b^2} + \frac{a d^2}{b^3} - \frac{a d^3}{b^4} \pm \&c. \dots$
 6. " $\frac{a}{b} + \frac{a d}{b^2} + \frac{a d^2}{b^3} + \frac{a d^4}{b^4} + \&c. \dots$

d. ANSWERS TO THE EXERCISES IN POWERS OF POWERS.

- | | |
|--|----------------------------------|
| 1. Ans. a^4 . | 2. Ans. a^6 . |
| 3. " a^{12} . | 4. " $4 \cdot a^4$. |
| 5. " $125 a^3 b^6$. | 6. " $16 a^{12} b^8$. |
| 7. " $9 a^4 b^4 c^4$. | 8. " a^9 . |
| 9. " $+ a^4$. | 10. " $- a^6$. |
| 11. " $+ a^8$. | |
| 12. " $- a^{10}, + a^{12}, - a^{14}, + a^{16}$, respectively. | |
| 13. " $64 a^6 b^9 c^3$. | 14. Ans. $- 8 a^6 b^9 c^6$. |
| 15. " $- 64 a^6 b^9 c^9$. | 16. " $a^{15} b^{20} c^{25}$. |
| 17. " $+ 729 a^{12} b^6$. | 18. " $16 a^4 b^6 c^8 f^2 g^2$. |
| 19. " $9 a^4$. | 20. " $8 a^6 b^9$. |
| 21. " $+ 4 a^4$. | 22. " $- 8 a^6$. |
| 23. " $4096 a^{12}$. | 24. " $+ a^{10}$. |
| 25. " $+ b^{-12}$. | 26. " $4096 a^{36}$. |
| 27. " a^{120} . | 28. " $+ a^{60}$. |
| 29. " $- a^{-3}$. | 30. " $+ a^{-4}$. |
| 31. " $- a^{-10}$. | 32. " $+ a^{-12}$. |
| 33. " $- a^{15}$. | |
| 34. " $+ 2^{-10} a^{-10} = \frac{1}{1024 a^{10}}$. | |
| 35. " $+ 2^{-4} a^8 = \frac{a^8}{32}$. | |
| 36. " $- 2^{15} a^{15} = - 32768 a^{15}$. | |

SECTION III.

EXERCISES IN ADDITION, SUBTRACTION,
MULTIPLICATION, DIVISION, AND
RAISING TO POWERS OF
FRACTIONS.*A. Addition.*A. ANSWERS TO THE EXERCISES IN ADDITION OF SIMPLE
QUANTITIES.

1. Ans. $\frac{a + 2a + 3a}{b} = \frac{6a}{b}.$

2. " $\frac{9b}{2c}.$

3. Ans. $\frac{9a}{4}.$

4. " $\frac{11}{12}a^2.$

5. " $\frac{1}{4}a^2 - 4b^3.$

6. " $\frac{3a^2}{b} - \frac{a^2}{b^2}.$

7. " $\frac{9a^3}{b^4} - a^4.$

8. " $-\frac{2a^3}{x^3} + \frac{7a^4}{5}.$

9. " $\frac{11}{4}a^4.$

10. " $\frac{205}{58}ab^2x^3.$

11. " $\frac{ad + bc}{bd}.$

12. " $\frac{2ad + 2bc}{2bd}.$

13. " $\frac{acd + b^2d + bc^2}{bcd}.$

14. " $\frac{6acd + 3b^2d + 2c^2b}{6bcd}.$

15. " $\frac{6acd + 3b^2d + 2bc^2}{12bcd}.$

16. Ans. $\frac{40 a c d + 45 b^2 d + 48 b c^2}{60 b c d}$.
17. " $\frac{40 a d + 45 b d + 48 c^2}{60 b c d}$.
18. " $\frac{b c + a c - a b}{a b c}$.
19. " $\frac{6 b c + 4 a c + 3 a b}{12 a b c}$.
20. " $\frac{a b e f - b c f + c d e}{c d e f}$.
21. " $\frac{9 a d + 6 b - 2 c}{3 b c d}$.
22. " $\frac{a c f h + b^2 f h + b c d h + b c f g}{b c f h}$.
23. " $\frac{12 a d e g + 6 b c e g + 4 b d^2 g + 3 b d e f}{12 b d e g}$.
24. " $\frac{c d f + b d f + b c f + b c d + b c d f g + b c d f h}{b c d f}$.

b. ANSWERS TO THE EXERCISES IN ADDITION OF COMPOUND QUANTITIES.

1. Ans. $\frac{7}{2} a - \frac{27}{10} c - \frac{1}{2} d = 3\frac{1}{2} a - 2\frac{7}{10} c - 4\frac{1}{2} d$.
2. " $\frac{2}{3} f - \frac{17}{10} a = 2\frac{1}{3} f - 2\frac{7}{10} a$.
3. " $8\frac{1}{10} a - \frac{1}{10} b + 7 c - f$.
4. " $1\frac{1}{2} a + 1\frac{2}{15} b$.
5. " $\frac{1}{2} + \frac{3}{2b} + \frac{3}{bf} + \frac{11}{6ab}$.
6. " $\frac{a}{b} + \frac{3c}{2d} + \frac{11e}{6f} + \frac{11g}{6h}$.

7. Ans. $\frac{c^2}{2d^2} + \frac{7e^3}{12f^3} + \frac{2g^4 + g^3}{4h^4}$.
8. " $\frac{2a}{3} + \frac{23bc}{20} + \frac{71de}{35} - \frac{fgh}{3}$.
9. " $\frac{2b^2c^3 + 4c^3d + 21a^2c^3}{14g^2h^3} + \frac{f^3g - 3f^2g - l^3m}{8m^3n^3}$.
10. " $\frac{abc^2 + ab^2c + ab^3}{c^3} + \frac{c^3d - c^2de^2 - cde^4}{e^5}$
 $+ \frac{efg^5 + e^4fg^2 - e^5f^2}{g^6}$.

B. Subtraction.

a ANSWERS TO THE EXERCISES IN SUBTRACTION OF SIMPLE QUANTITIES.

1. Ans. $8\frac{3}{4}$. 2. Ans. $-16\frac{1}{4}$.
3. " $-1\frac{8}{15}$. 4. " $6\frac{8}{15}$.
5. " $\frac{1}{3}$. 6. " $-\frac{3}{8}$.
7. " $\frac{2}{15}a$. 8. " $-\frac{5}{12}b$.
9. " $\frac{7}{12}b = 1\frac{5}{12}b$. 10. " $-\frac{2}{3}b$.
11. Ans. $\frac{ad - bc}{bd}$.
12. " $\frac{adf + bcf - bde}{bdf}$.
13. " $\frac{2ad - 3bc}{4bd}$.
14. " $\frac{10adf - 15bcf - 16bde}{20bdf}$.

b. ANSWERS TO THE EXERCISES IN COMPOUND SUBTRACTION.

1. Ans. $6\frac{1}{2}a - 1\frac{1}{8}b + 6\frac{1}{3}d$.
2. " $12\frac{8}{10}a - 22\frac{5}{11}b - 23\frac{2}{5}d$.
3. " $\frac{2}{3}a + 8\frac{1}{8}d + 3\frac{1}{2}c - 1\frac{1}{4}g$.
4. " $8\frac{1}{4}a - 5\frac{1}{2}l - \frac{1}{2}m$.
5. " $1\frac{1}{4}a^2 + \frac{2}{3}a^2b^2c + \frac{1}{2}a^2b^2c^2 - \frac{2}{3}ab^3$.
6. " $\frac{2}{3}a^3 + a^4d + \frac{2d^3}{e} + \frac{2e}{3f}$.
7. " $\frac{1}{2a} + \frac{1}{2b}$.
8. " $\frac{a}{2b} + \frac{3c}{2d}$.
9. " $\frac{3ab + mn}{3cd} + \frac{4ef - ap}{4gh}$.
10. " $\frac{2ab - a}{4cd} + \frac{3h - gh}{3mn}$.
11. " $\frac{a^2 - 1}{a} + \frac{b^2 - 1}{b}$.
12. " $\frac{c^2d^2 - b}{cd} + \frac{f^2g^2 + e}{fg} + \frac{h^2 - k}{h}$.

C. *Multiplication.*a. ANSWERS TO THE EXERCISES IN MULTIPLICATION OF
SIMPLE QUANTITIES.

1. Ans. $\frac{2}{3}a$.
2. " $\frac{8}{3}a = 2\frac{2}{3}a$.
3. " $\frac{51}{4}ab = 12\frac{3}{4}ab$.

4. Ans. $1\frac{1}{8}af = 1\frac{1}{8}af$. 5. Ans. $-\frac{1}{4}ab$.
 6. " $\frac{1}{2}\frac{1}{8}ab$. 7. " $-\frac{abc d^2 e}{8}$.
 8. " $\frac{3c^2 dfg}{32}$. 9. " $86\frac{1}{4}deg$.
 10. " $\frac{1}{8}abcd = \frac{abcd}{6}$.
 11. " $\frac{3}{15}abdef$. 12. " $\frac{1}{3}\frac{2}{5}abcdef$.
 13. " -1 . 14. " $\frac{3h^2 l}{5gm^2}$.
 15. " $\frac{1}{ab}$. 16. " $\frac{ac}{bd}$.
 17. " $\frac{1}{abc}$. 18. " $\frac{ace}{bdf}$.
 19. " $\frac{2ace}{5bdf}$. 20. " $\frac{4}{5}a^2 ab d^2 f$.
 21. " $\frac{7ab d^2 f}{8}$. 22. " 3 .
 23. " $\frac{3c}{x}$. 24. " $\frac{3c^2}{x^2}$.
 25. " $\frac{4cd}{3abfg}$. 26. " $\frac{3c^2 d^4}{4a^2 bf^2}$.
 27. " $\frac{6a^6}{b^5}$. 28. " $\frac{3a^6 b^9}{5c^8 d^6}$.

b. ANSWERS TO THE EXERCISES IN COMPOUND
MULTIPLICATION.

1. Ans. $\frac{1}{2}ab - \frac{1}{4}b^2c - \frac{1}{15}bdf$.
 2. " $a^2d - \frac{2}{9}a d^2e - \frac{5}{15}adfg$.
 3. " $\frac{1}{4}a + \frac{2}{5}b - \frac{c}{6}$.

4. “ $\frac{b}{6} - \frac{c}{15} + \frac{d}{18}.$
5. “ $\frac{1}{2} c - \frac{3}{15} d + \frac{4}{5} e.$
6. “ $20 a - \frac{26 c b}{d} - \frac{24 h}{5}.$
7. “ $\frac{5 a^2}{8 b d} - \frac{3 a c}{10 d^2} + \frac{9 a h}{28 g d} + \frac{9 a}{2}.$
8. “ $-\frac{1}{4 b} + \frac{c}{12 a d} - \frac{2 h}{15 a g} + \frac{4}{15 a}.$
9. “ $\frac{1}{3} a^2 + \frac{37}{36} a b - \frac{1}{2} a c - \frac{5}{8} b^2 + \frac{5}{8} b c.$
10. “ $\frac{1}{3} a^2 - \frac{2}{15} a b - \frac{1}{15} b^2.$
11. “ $\frac{2}{3} a^4 - \frac{17}{120} a^2 b^2 + \frac{1}{4} b^4.$
12. “ $6\frac{2}{3} a^2 - 16\frac{1}{2} a b + 9\frac{3}{8} b^2.$
13. “ $\frac{1}{2} a^2 - \frac{1}{10} a b - \frac{1}{8} a c - \frac{2}{5} b^2 + \frac{1}{5} b c - \frac{7}{8} c^2.$
14. “ $5 x^4 + \frac{7}{2} a x^3 - \frac{2}{3} a^2 x^2 + \frac{7}{3} a^3 x - \frac{5}{4} a x^2$
 $- \frac{2}{3} a^2 x + \frac{7}{8} a^3.$
15. “ $\frac{1}{4} a^2 + \frac{1}{3} a b + \frac{1}{8} b^2 - \frac{1}{18} c^2.$
16. “ $\frac{1}{4} a^2 + \frac{1}{3} a b + \frac{1}{4} a c - \frac{1}{4} a d + \frac{1}{8} b^2 + \frac{1}{8} b c$
 $- \frac{1}{25} b d + \frac{1}{18} c^2 - \frac{1}{10} c d + \frac{1}{25} d^2.$

Division.

C. ANSWERS TO THE EXERCISES IN DIVISION OF SIMPLE QUANTITIES.

- $$\begin{array}{ll} 1. \text{ Ans. } \frac{a}{2b} & 2. \text{ Ans. } \frac{a}{6b} \\ 3. \text{ " } -\frac{a}{bc} & 4. \text{ " } \frac{ad}{bc} \\ 5. \text{ " } \frac{df}{e} & 6. \text{ " } \frac{fg}{h} \end{array}$$

- | | |
|-----------------------------------|----------------------------------|
| 7. " $\frac{6 a b d}{c}$. | 8. " $\frac{b}{a}$. |
| 9. " $\frac{b}{a}$. | 10. " $-\frac{2 a}{3 b^2}$. |
| 11. " $\frac{2 a b}{5 c d^3}$. | 12. " $\frac{2 f m^2}{g^2}$. |
| 13. " $\frac{2}{m n}$. | 14. " $-\frac{3 f^2 g}{a}$. |
| 15. " $\frac{9 c}{10 b d}$. | 16. " $\frac{1}{2 g}$. |
| 17. " $2 f^2 g^3$. | 18. " $\frac{1}{2 g^3}$. |
| 19. " $\frac{3 a^2 b^4}{7 c^3}$. | 20. " $\frac{b}{a}$. |
| 21. " $\frac{3 b}{2 a}$. | 22. " $-\frac{5 c^3}{4 a^2 b}$. |
| 23. " $\frac{9 d}{2 a b^3}$. | 24. " $-\frac{5 c d}{a^2 b}$. |
| 25. " $-\frac{6}{a^2 b^3}$. | 26. " $2 b c^4$. |

*E. Answers to the Exercises in Reductions of
Fractions to the Lowest Term.*

- | | |
|------------------------------|-------------------------------|
| 1. Ans. $\frac{2 a}{3 b}$. | 2. Ans. $\frac{2 a^2}{3 c}$. |
| 3. " $\frac{3 a^2}{4 b^2}$. | 4. " $\frac{2 a}{c}$. |
| 5. " $\frac{x^3}{3 y^2}$. | 6. " $\frac{2}{3 y}$. |

- | | |
|---------------------------|-----------------------------|
| 7. Ans. $\frac{a}{3}$. | 8. Ans. $\frac{31a}{6b}$. |
| 9. " $\frac{2a}{3bc^2}$. | 10. " $\frac{6c^2}{7a^3}$. |
| 11. " $\frac{1}{2ab}$. | 12. " $\frac{a}{2b}$. |
| 13. " $\frac{4}{5a}$. | 14. " $\frac{a}{2bc}$. |
| 15. " $\frac{1}{xy}$. | 16. " $\frac{1}{xy}$. |
| 17. " $2a^2y^2$. | 18. " $5ab$. |
| 19. " $4acd$. | 20. " $\frac{2}{3}abcd$. |
-

F. Answers to the Exercises in Raising Fractions to Powers.

- | | |
|--|----------------------------------|
| 1. Ans. $\frac{a^2}{b^2}$. | 2. Ans. $\frac{9a^2}{16b^2}$. |
| 3. " $\frac{27a^6}{64b^9}$. | 4. " $\frac{1}{a^2}$. |
| 5. " $+\frac{1}{a^8}$. | 6. " $\frac{1}{9a^4b^{10}}$. |
| 7. " $\frac{1}{8a^6}$. | 8. " $-\frac{27}{64f^6}$. |
| 9. " $+\frac{16a^8b^4}{81}$. | 10. " $\frac{64}{27a^6}$. |
| 11. " $+\frac{a^2}{b^2}$. | 12. " $+\frac{a^{12}}{b^{12}}$. |
| 13. " $+\frac{a^{-12}}{b^{-12}} = \frac{b^{12}}{a^{12}}$. | 14. " $\frac{a^4b^8}{c^8d^4}$. |
- 3*

$$15. \text{ Ans. } \frac{d^{16} b^{20}}{c^{12} a^4 f^4}.$$

$$16. \text{ Ans. } \frac{27 a^9 b^6}{64 d^{12}}.$$

$$17. \text{ " } + \frac{1}{9 a^4 e^2}.$$

$$18. \text{ " } - \frac{27}{64 a^6 b^6}.$$

$$19. \text{ " } + \frac{5^{-4}}{6^{-4} a^{-12} b^{-12}}.$$

$$20. \text{ " } + \frac{4 a^4 b^6}{9 c^6 d^6}.$$

SECTION IV.

OF ROOTS.

A. Of Square Roots.

a. ANSWERS TO THE EXERCISES IN THE EXTRACTION OF
SQUARE ROOTS OF NUMBERS.

- | | |
|-------------------|-------------------|
| 1. Ans. 4. | 2. Ans. 10. |
| 3. " 11. | 4. " 16. |
| 5. " 24. | 6. " 64. |
| 7. " 98. | 8. " 247. |
| 9. " 763 | 10. " 504. |
| 11. " 194. | 12. " 950. |
| 13. " 7563. | 14. " 8276. |
| 15. " 5083. | 16. " 15367. |
| 17. " 40093. | 18. " 279433. |
| 19. " 37695. | 20. " 203975. |
| 21. " 40005. | 22. " 6950078. |
| 23. " 3476905. | 24. " 2.23606. |
| 25. " 3.6055. | 26. " 4.69041. |
| 27. " 9.79795. | 28. " 12.36931... |
| 29. " 10.04987... | 30. " 2.76586... |
| 31. " 3.09838... | 32. " 3.90357... |

33. Ans. 0 : 23664 . . .	34. Ans. 0.08882 . . .
35. " 0.05477 . . .	36. " 0.11832 . . .
37. " $\frac{1}{2}$.	38. " $\frac{3}{4}$.
39. " $\frac{8}{9}$.	40. " $\frac{15}{16}$.
41. " $\frac{11}{12}$.	42. " $\frac{15}{16}$.
43. " $\frac{123}{124}$.	44. " $\frac{458}{459}$.
45. " $\frac{1238}{1239}$.	46. " $\frac{18452}{18453}$.
47. " 1.32287 . . .	48. " 1.24721 . . .
49. " 1.80277 . . .	50. " 3.41869 . . .
51. " 2.92575 . . .	52. " 2.71313.
53. " 2.88203.	54. " 1.29099 . . .
55. " 0.89442 . . .	56. " 1.29099 . . .
57. " 0.93541 . . .	58. " 0.64549 . . .
59. " 0.24253 . . .	60. " 0.54772 . . .

b. ANSWERS TO THE EXERCISES IN THE EXTRACTION OF THE
SQUARE ROOTS OF ALGEBRAIC QUANTITIES.

1. Ans. a .	2. Ans. a^2 .
3. " a^3 .	4. " a^5 .
5. " $2a^4$.	6. " $4ab^2$.
7. " $8a^2b^3$.	8. " $10a^3b^3$.
9. " $9abc$.	10. " $11a^2bc^3$.
11. " $16a^2b^8$.	12. " $12xg^3z^4$.
13. " $3a^2bf^6g^4$.	14. " $\frac{a}{b}$.
15. " $\frac{ab}{c^2}$.	16. " $\frac{2a}{b}$.
17. " $\frac{3a}{4b^2}$.	18. " $\frac{2ab}{3c^2d^4}$.

19. Ans. $\frac{1}{a}$.

20. Ans. $\frac{1}{2a}$.

21. " $\frac{1}{3ab^2}$.

22. " $\frac{1}{3x}$.

23. " $\frac{2}{3ab}$.

24. " $\frac{6a^3}{7b}$.

25. " $\frac{1}{4}a^3$.

26. " $\frac{1}{4}a^2bc$.

27. " $a + b$.

28. " $a - b$.

29. " $a - \frac{b}{2}$.

30. " $x + 1$.

31. " $x - 1$.

32. " $f^3 + 3x^4$.

33. " $f^3 - 3x^4$.

34. " $\frac{3}{2}a^4 + \frac{2}{3}n^3$.

35. " $\frac{3}{2}a^4 - \frac{2}{3}n^3$.

36. " $\frac{5}{2}ab - \frac{1}{3}c^2$.

37. " $x^2 - \frac{1}{2}ax$.

38. " $a^2 + b^2$.

39. " $a^4 + b^4$.

40. " $x^5 + y^5$.

41. " $\frac{a}{b} - \frac{2b}{3c}$.

42. " $\frac{a}{b} - \frac{c}{d}$.

43. " $a + b + c$.

44. " $3x - 5a + \frac{a^2}{2}$.

45. Ans. $2x^2 + 2ax + 4b^2$.

46. " $3a - b + 5c + d$.

47. " $\frac{3}{2} + 2x - 7x^2$.

48. " $3x^2 - \frac{ax}{2} + bx$.

49. " $\frac{a+b}{x-a}$.

50. " $\frac{a-2b}{x^2+3a^2}$.

*B. Cube Roots.**a. ANSWERS TO THE CUBE ROOTS OF NUMBERS.*

1. Ans. 2.	2. Ans. 4.
3. " 5.	4. " 7.
5. " 9.	6. " 10.
7. " 12.	8. " 24.
9. " 23.	10. " 96.
11. " 74.	12. " 55.
13. " 108.	14. " 135.
15. " 223.	16. " 80.
17. " 106.	18. " 258.
19. " 368.	20. " 343.
21. " 401.	22. " 200.
23. " 420.	24. " 683.
25. " 698.	26. " 1854.
27. " 1936.	28. " 4820.
29. " 4865.	30. " 2667.
31. " 2009.	32. " 1.9129...
33. " 2.28942...	34. " 4.3444...
35. " 6.4392...	36. " 8.1981...
37. " 8.8237...	38. " 1.7967...
39. " 2.1897...	40. " 4.6856...
41. " 3.0455...	42. " 1.78668...
43. " $\frac{1}{2}$.	44. " $\frac{1}{3}$.
45. " 0.4622...	46. " $\frac{2}{3}$.
47. " $\frac{7}{8}$.	48. " $\frac{5}{6}$.
49. " $7\frac{1}{2}$.	50. " $37\frac{1}{2}$.
51. " 0.8735...	52. " 0.94103...

- | | |
|----------------------|----------------------|
| 53. Ans. 0.92831 ... | 54. Ans. 0.79370 ... |
| 55. " 0.69336 ... | 56. " 1.14471 ... |
| 57. " 1.48124 ... | 58. " 0.70949 ... |
| 59. " 1.56049 ... | 60. " 2.50222 ... |
-

b. ANSWERS TO THE EXTRACTION OF CUBE ROOTS OF
ALGEBRAIC QUANTITIES.

- | | |
|-------------------------------------|-------------------------------------|
| 1. Ans. b . | 2. Ans. b^3 . |
| 3. " a^3 . | 4. " $-3ab^3$. |
| 5. " $5a^2bc^3$. | 6. " $7ab^2c^4$. |
| 7. " $-\frac{a}{b}$. | 8. " $\frac{2a}{3c^2}$. |
| 9. " $\frac{1}{a}$. | 10. " $-\frac{1}{3ab}$. |
| 11. " $-\frac{1}{xy}$. | 12. " abc^3d^2 . |
| 13. " $-\frac{a^2}{b^3}$. | 14. " $a+b$. |
| 15. " $a-b$. | 16. " $x+2$. |
| 17. " $x+1$. | 18. " $2a-7x$. |
| 19. " x^2-2cx . | 20. " $2-x^5$. |
| 21. " $\frac{a}{b} + \frac{c}{d}$. | 22. " $\frac{1}{a} - \frac{1}{b}$. |
| 23. " $a+b+c$. | 24. " x^2+x+1 . |
-

c. ANSWERS TO THE HIGHER ROOTS OF SIMPLE QUANTITIES.

- | | |
|------------|------------|
| 1. Ans. 2. | 2. Ans. 3. |
| 3. " 8. | 4. " 12. |

- | | |
|------------------------------|---------------------------------|
| 5. Ans. 25. | 6. Ans. 4. |
| 7. " 5. | 8. " $4 a b^2$. |
| 9. " $\frac{a^2 b}{3 c^3}$. | 10. " $\frac{1}{2 a^2}$. |
| 11. " $3 a^2 b^3 c$. | 12. " $\frac{2 a^2}{b^3 c^4}$. |
-

D. Addition, Subtraction, Multiplication, and Division of Roots.

a. ANSWERS TO THE EXERCISES IN ADDITION OF ROOTS.

- | | |
|---|---|
| 1. " $2\sqrt{a}$. | 2. " $2\sqrt{2}$. |
| 3. " $7\sqrt{3}$. | 4. " $6\sqrt{b}$. |
| 5. " $2\sqrt{2}$. | 6. " $2\frac{3}{4}\sqrt{5}$. |
| 7. " $2\sqrt{3} - \frac{1}{2}\sqrt{5}$. | 8. " $1\frac{1}{2}\sqrt{\frac{3}{2}} = \frac{3}{2}\sqrt{\frac{3}{2}}$. |
| 9. " $3\sqrt[3]{a}$. | 10. " $4\frac{1}{2}\sqrt[3]{a}$. |
| 11. " $4\sqrt[4]{a^3}$. | 12. " $\frac{5}{2}\sqrt[3]{2}$. |
| 13. " $2\frac{3}{4}\sqrt[3]{\frac{3}{2}}$. | 14. " $3\sqrt[3]{3} + 3\sqrt{3}$. |
| 15. " $2\sqrt[5]{a}$. | 16. " $4\sqrt[3]{4} - 3\sqrt[3]{3}$. |
| 17. " $\frac{5}{8}\sqrt{\frac{1}{2}}$. | 18. " $2\frac{1}{10}\sqrt{2}$. |
| 19. " $\sqrt[3]{7}$. | 20. " $\frac{1}{3}\sqrt[3]{9} + \frac{1}{4}\sqrt{5}$. |
| 21. " $16\sqrt{2} - 4\sqrt[3]{7}$. | 22. " $17\sqrt{2}$. |
| 23. " $11\sqrt{2} - 5\sqrt[3]{6} + 5\sqrt[4]{5}$. | |
| 24. " $5\frac{3}{4}\sqrt{a} - \frac{1}{4}\sqrt{ab} - 1\frac{1}{8}\sqrt[3]{b} = 2\frac{3}{4}\sqrt{a} - \frac{1}{4}\sqrt{ab}$
$- \frac{1}{8}\sqrt[3]{b}$. | |

b. ANSWERS TO THE EXERCISES IN SUBTRACTION.

- | | |
|---|-----------------------------------|
| 1. Ans. $\sqrt{3}$. | 2. Ans. $3\sqrt{a}$. |
| 3. " $\frac{1}{2}\sqrt{ab}$. | 4. " $2\sqrt[3]{a}$. |
| 5. " $\frac{1}{2}\sqrt[3]{2}$. | 6. " $4\sqrt[3]{\frac{1}{2}}$. |
| 7. " $-7\sqrt{2}$. | 8. " $5\sqrt{3}$. |
| 9. " $-\sqrt{2}$. | 10. " $-\frac{1}{2}\sqrt[3]{3}$. |
| 11. " $12\sqrt{7} - 3\sqrt{6} + \frac{2}{3}\sqrt[3]{11}$. | |
| 12. " $7\sqrt{6} - 5\sqrt[3]{2} + \frac{2}{3}\sqrt[4]{5}$. | |
| 13. " $\sqrt{a} + 8\sqrt{b}$. | |
| 14. " $7\sqrt{ab} + \frac{1}{2}\sqrt{bc}$. | |
| 15. " $\sqrt[4]{a} + 5\sqrt[3]{b}$. | |
| 16. " $5\sqrt[4]{a^3} + 2\sqrt[3]{a^2b} - 9\sqrt{c}$. | |
-

c. ANSWERS TO THE ABBREVIATIONS AND TRANSFORMATIONS

- | | |
|--|--|
| 1. Ans. $5\sqrt{6}$. | 2. Ans. $13\sqrt{2}$. |
| 3. " $5\sqrt{3}$. | 4. " $-10\sqrt{2}$. |
| 5. " $2\frac{1}{2}\sqrt{3} = \frac{5}{2}\sqrt{3}$. | 6. " $5\frac{1}{2}\sqrt{3} = \frac{11}{2}\sqrt{3}$. |
| 7. " $-8\sqrt[3]{2}$. | 8. " $21\sqrt[3]{2}$. |
| 9. " $5\sqrt[3]{2}$. | 10. " $\frac{2}{3}\sqrt{2}$. |
| 11. " $\frac{1}{2}\sqrt{2}$. | |
| 12. " $\frac{1}{4}\sqrt{2} = \sqrt{2}, (\frac{1}{2}\sqrt{\frac{1}{2}} \text{ being equal to } \frac{1}{2} \cdot \frac{1}{2}\sqrt{2}).$ | |

d. MULTIPLICATION OF ROOTS.

1. ANSWERS TO THE MULTIPLICATION OF SIMPLE QUANTITIES.

- | | |
|---|--|
| 1. Ans. 2. | 2. Ans. 3. |
| 3. " $\sqrt{10}$. | 4. " $2\sqrt{30}$. |
| 5. " $6\sqrt{30}$. | 6. " $42\sqrt{12}$. |
| 7. " a . | 8. " b . |
| 9. " \sqrt{ab} . | 10. " $b\sqrt{ac}$. |
| 11. " $ac\sqrt{bd}$. | 12. " acd . |
| 13. " a . | 14. " b . |
| 15. " $\sqrt[3]{ab}$. | 16. " $ac\sqrt[3]{b}$. |
| 17. " $ac\sqrt[3]{bd}$. | 18. " $15ab^2$. |
| 19. " $15\sqrt[3]{72}$. | 20. " $12\sqrt[3]{4}$. |
| 21. " $45\sqrt{3}$. | 22. " $\sqrt{\frac{8}{15}}$. |
| 23. " $\sqrt{\frac{15}{4}} = \frac{1}{2}\sqrt{15}$. | 24. " $3\sqrt{3}$. |
| 25. " $12\sqrt{\frac{1}{3}}$. | 26. " $\frac{2}{3}\sqrt{\frac{8}{15}}$. |
| 27. " $\frac{ae}{bf}\sqrt{\frac{cg}{dh}}$. | 28. " $\sqrt{\frac{1}{ab}}$. |
| 29. " 1, (because $\sqrt{\frac{1}{a}} \times \sqrt{a} = \sqrt{\frac{a}{a}} = \sqrt{1}$.) | |
| 30. " 1. | |
| 31. " 12. | |
| 32. " $\frac{1}{ac}\sqrt{\frac{1}{bd}} = \frac{1}{ac\sqrt{bd}}$. | |
| 33. " $\frac{1}{6}\sqrt{\frac{1}{12}} = \frac{1}{6\sqrt{12}}$. | |
| 34. " $\frac{1}{8}\sqrt{\frac{1}{15}} = \frac{1}{8\sqrt{15}}$. | |

$$35. \text{ Ans. } \sqrt[3]{\frac{a^3}{b^3}}.$$

$$36. \text{ Ans. } \sqrt[3]{\frac{a c}{b d}}.$$

$$37. \text{ " } a d \sqrt[3]{\frac{b e}{c f}}.$$

$$38. \text{ " } \frac{a e}{b f} \sqrt[3]{\frac{c g}{d h}}.$$

$$39. \text{ " } \sqrt[3]{\frac{1}{a b}} = \frac{1}{\sqrt[3]{a b}}.$$

$$40. \text{ " } \sqrt[3]{a}.$$

$$41. \text{ " } a b.$$

$$42. \text{ " } a c \sqrt[3]{\frac{1}{b d}} = \frac{a c}{\sqrt[3]{b d}}.$$

$$43. \text{ " } \frac{1}{a d} \sqrt[3]{\frac{1}{b f}} = \frac{1}{a d \sqrt[3]{b f}}.$$

$$44. \text{ " } \sqrt[3]{\frac{1}{6}} = \frac{1}{\sqrt[3]{6}}.$$

$$45. \text{ " } \sqrt[3]{\frac{2}{18}} = \sqrt[3]{\frac{1}{9}} = \frac{1}{\sqrt[3]{9}}.$$

$$46. \text{ " } \sqrt[3]{\frac{1}{3}} = 5, \text{ (because } \sqrt[3]{\frac{1}{3}} \times \sqrt[3]{3} = \sqrt[3]{\frac{1}{1}} \text{.)}$$

$$47. \text{ " } \sqrt[4]{a^2} = \sqrt{a}.$$

$$48. \text{ Ans. } \sqrt[4]{a b^2 c}.$$

$$49. \text{ " } 12 \sqrt[4]{30}.$$

$$50. \text{ " } \frac{1}{8} \sqrt[4]{18}.$$

$$51. \text{ " } \sqrt[4]{a b c d}.$$

$$52. \text{ " } a b^2 c e \sqrt[4]{d}.$$

$$53. \text{ " } 12 a b.$$

$$54. \text{ " } 4 \sqrt[3]{a^2 b c^2}.$$

$$55. \text{ " } \sqrt[3]{a b c^2 d e^2 f}.$$

$$56. \text{ " } \sqrt[3]{\frac{1}{b^3}} = \frac{1}{b}.$$

$$57. \text{ " } \frac{3}{f}.$$

$$58. \text{ Ans. } \sqrt{\frac{1}{cf^2}} = \frac{1}{\sqrt{cf^2}} = \frac{1}{f\sqrt{c}}.$$

$$59. \text{ " } \frac{1}{b}.$$

$$60. \text{ " } \frac{2}{3}\sqrt{\frac{3}{2}} = 1\sqrt{1} = 1.$$

2. ANSWERS TO THE MULTIPLICATION OF COMPOUND QUANTITIES.

$$1. \text{ Ans. } 3\sqrt{2} + \sqrt{10}. \quad 2. \text{ Ans. } 6\sqrt{2} + 8\sqrt{5}.$$

$$3. \text{ " } 6 + 4\sqrt{10}. \quad 4. \text{ " } 3\sqrt{3} + \sqrt{15} - 6.$$

$$5. \text{ " } 6 - 5 = 1.$$

$$6. \text{ " } 2 - \sqrt{3} - 3 = -\sqrt{3} - 1.$$

$$7. \text{ " } 3 - 17\sqrt{6}. \quad 8. \text{ Ans. } 42 - 13\sqrt{6}.$$

$$9. \text{ " } 3 - 2\sqrt{2}. \quad 10. \text{ " } -3 + 3\sqrt{7}.$$

$$11. \text{ " } 24\frac{1}{2}. \quad 12. \text{ " } 41.$$

$$13. \text{ " } 1 + \sqrt{6}. \quad 14. \text{ " } 75 + 11\sqrt{15}.$$

$$15. \text{ " } 5 + 2\sqrt{6}. \quad 16. \text{ " } a^2 - b.$$

$$17. \text{ " } a + 2\sqrt{ab} + b.$$

$$18. \text{ " } c^2a - cd\sqrt{ab} + ac\sqrt{ab} - adb.$$

$$19. \text{ " } -62. \quad 20. \text{ Ans. } 87 + 12\sqrt{42}.$$

$$21. \text{ " } 87 - 12\sqrt{42}. \quad 22. \text{ " } \sqrt[3]{4} - 4\sqrt[3]{9}.$$

$$23. \text{ " } \sqrt[3]{\frac{1}{4}} - \sqrt[3]{\frac{1}{8}}. \quad 24. \text{ " } \frac{a}{b} - \frac{c}{d}.$$

c. ANSWERS TO THE EXERCISES IN DIVISION.

$$1. \text{ Ans. } 1.$$

$$2. \text{ Ans. } \sqrt{a^2} = a.$$

$$3. \text{ " } \sqrt{\frac{a}{b}}.$$

$$4. \text{ " } 2\sqrt{a^2} = 2a.$$

5. Ans. $3a$. 6. Ans. $2\sqrt{\frac{a}{b}}$.
7. " $\frac{3}{4}\sqrt{\frac{b}{c}}$. 8. " $\frac{a}{c}\sqrt{\frac{b}{d}}$.
9. " \sqrt{a} . 10. " \sqrt{a} .
11. " $2\sqrt{ab^2} = 2b\sqrt{a}$. 12. " $\sqrt{a^2b^2} : \sqrt{ab} = ab$.
13. " $\sqrt{\frac{ad}{bc}}$. 14. " $2\sqrt{\frac{ad}{bc}}$.
15. " $\frac{mq}{np}\sqrt{\frac{ad}{bc}}$. 16. " $\sqrt{\frac{b}{a}}$.
17. " $\sqrt{\frac{3}{2}} = \sqrt{\frac{6}{4}} = \sqrt{\frac{1}{4}} \cdot 6 = \frac{1}{2}\sqrt{6}$.
18. " $\sqrt{\frac{5}{6}}$.
19. " $\frac{3}{5}\sqrt{\frac{5}{6}}$.
20. " $\frac{5}{6}\sqrt{\frac{3}{2}} = \frac{5}{6}\sqrt{\frac{6}{4}} = \frac{5}{12}\sqrt{6}$.
21. " 1. 22. Ans. $\sqrt[3]{a}$.
23. " $\sqrt[3]{\frac{a}{b}}$. 24. " $2\sqrt[3]{a}$.
25. " $2\sqrt[3]{a}$. 26. " $2\sqrt[3]{\frac{a^2}{b}}$.
27. " $\frac{2}{5}\sqrt[3]{\frac{b}{c}}$. 28. " $\frac{a}{c}\sqrt[3]{\frac{b}{d}}$.
29. " $\sqrt[3]{a^2}$. 30. " $\sqrt[3]{a^3} : \sqrt[3]{a} = \sqrt[3]{a^2}$.
31. " $3\sqrt[3]{a^2b}$.
32. " $\sqrt[3]{a^3b^3} : \sqrt[3]{ab} = \sqrt[3]{a^2b^2}$.
33. " $\sqrt[3]{\frac{ad}{bc}}$. 34. Ans. $2\sqrt[3]{\frac{ad}{bc}}$.

$$35. \text{ Ans. } \frac{m}{n} \frac{q}{p} \sqrt[3]{\frac{ad}{bc}} \quad 36. \text{ Ans. } \sqrt[3]{\frac{b}{a}}$$

$$37. \text{ " } \sqrt[3]{\frac{3}{2}} \quad 38. \text{ " } \sqrt[3]{\frac{5}{6}}$$

$$39. \text{ " } \frac{3}{5} \sqrt[3]{\frac{5}{6}} \quad 40. \text{ " } \frac{5}{6} \sqrt[3]{\frac{3}{2}}$$

E. Fractional Exponents.

$$1. \text{ Ans. } a^{\frac{1}{2}}.$$

$$2. \text{ Ans. } a^{\frac{2}{3}}$$

$$3. \text{ " } a^{\frac{2}{3}}.$$

$$4. \text{ " } a^{\frac{3}{4}}.$$

$$5. \text{ " } a^{\frac{3}{6}}.$$

$$6. \text{ " } a^{\frac{3}{7}}.$$

$$7. \text{ " } a^{\frac{1}{3}}.$$

$$8. \text{ " } a^{\frac{1}{4}}.$$

$$9. \text{ " } a^{\frac{5}{6}} = \sqrt[6]{a^5}.$$

$$10. \text{ " } a^{\frac{3}{4}} = \sqrt[4]{a^3}.$$

$$11. \text{ " } a^{\frac{7}{6}} = \sqrt[6]{a^7}.$$

$$12. \text{ " } a^{\frac{17}{12}} = \sqrt[12]{a^{17}}.$$

$$13. \text{ " } a^{\frac{9}{10}} = \sqrt[10]{a^9}.$$

$$14. \text{ " } a^{\frac{11}{6}} = \sqrt[6]{a^{11}}.$$

$$15. \text{ " } a^{\frac{17}{12}} = \sqrt[12]{a^{17}}.$$

$$16. \text{ " } a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a.$$

$$17. \text{ " } a^{\frac{8}{6}} = a^{\frac{4}{3}} = \sqrt[3]{a^4}.$$

$$18. \text{ " } a^{\frac{9}{6}} = a^{\frac{3}{2}} = \sqrt{a^3}. \quad 19. \text{ Ans. } a^{\frac{1}{6}} = \sqrt[6]{a}.$$

$$20. \text{ " } a^{\frac{1}{12}} = \sqrt[12]{a}.$$

$$21. \text{ " } a^{\frac{3}{6}} : a^{\frac{2}{6}} = a^{\frac{1}{6}} = \sqrt[6]{a}.$$

$$22. \text{ " } a^1 = \sqrt{a}$$

23. Ans. $a^{\frac{1}{16}} = \sqrt[16]{a}$. 24. Ans. $a^{\frac{1}{12}} = \sqrt[12]{a^{11}}$.
 25. " $a^{\frac{2}{3}} b^{\frac{1}{3}} c^{\frac{4}{3}}$. 26. " $a^{\frac{3}{4}} b^{\frac{2}{4}} c^{\frac{1}{4}}$.
 27. " $a^{\frac{2}{3}} c^{\frac{1}{3}}$. 28. " $a^{\frac{3}{4}} c^{\frac{2}{4}}$.
 29. " $a^{\frac{5}{4}} b^{\frac{4}{4}} = \sqrt[4]{a^5 b^4} = \sqrt[4]{a \cdot a^4 b^4} = a b \sqrt[4]{a}$.
 30. " $a^{\frac{1}{2}} b^{\frac{1}{2}} = \sqrt[12]{a^{17} b^{10}} = a \sqrt[12]{a^5 b^{10}}$.
 31. " $a^{\frac{1}{2}} b^{\frac{1}{2}} = \sqrt[4]{a b}$.
 32. " $a^{\frac{2}{3}} b^{\frac{5}{3}} : a^{\frac{1}{2}} b^{\frac{8}{3}} = a^{\frac{1}{6}} b^{\frac{1}{6}} = \sqrt[6]{a b}$.
-

F. Powers and Roots of Roots.

- | | |
|--|--|
| 1. Ans. a . | 2. Ans. b . |
| 3. " $a b$. | 4. " 2. |
| 5. " 5. | 6. " 3. |
| 7. " $b^{\frac{8}{3}} = \sqrt[3]{b^8}$. | 8. " $a^{\frac{1}{2}} = \sqrt[12]{a^{12}} = a^6$. |
| 9. " $a^{\frac{9}{2}} = \sqrt[2]{a^9} = \sqrt[2]{a^8 \cdot a} = a^4 \sqrt[2]{a}$. | |
| 10. " $\sqrt[3]{a^{15}}$. | |
| 11. " $\sqrt[3]{a^4} = \sqrt[3]{a^3 \cdot a} = a \sqrt[3]{a}$. | |
| 12. " $\sqrt[3]{a^{12}} = a^4$. | 13. Ans. $a^{\frac{1}{4}} = \sqrt[4]{a}$. |
| 14. " $a^{\frac{1}{6}} = \sqrt[6]{a}$. | 15. " $a^{\frac{3}{4}} = \sqrt[4]{a^3}$. |
| 16. " $a^{\frac{5}{8}} = \sqrt[8]{a^5}$. | 17. " $a^{\frac{1}{12}} = \sqrt[12]{a}$. |
| 18. " $a^{\frac{2}{3}} = a^{\frac{1}{3}} = \sqrt[3]{a}$. | 19. " $\sqrt[4]{a^3}$. |
| 20. " $\sqrt[6]{a^7}$. | 21. " $\sqrt[6]{a^7}$. |

22. Ans. $\sqrt[3]{a}$.

23. Ans. $\sqrt[3]{a}$.

24. " $\sqrt[3]{a}$.

Note. When the pupil has once understood the reason why, in extracting the root of a root, we multiply the radical exponents, he need no longer write them in form of fractional exponents; but at once multiply the two or more radical exponents.

G. Imaginary Quantities.

ADDITION.

1. Ans. $6\sqrt{-1}$.

2. Ans. $3\sqrt{-1}$.

3. " $7\sqrt{-1} - \sqrt{3}$.

$$4. " \quad 7\sqrt{-1} + \sqrt{-4} = 7\sqrt{-1} + \sqrt{4 \cdot -1} \\ = 7\sqrt{-1} + 2\sqrt{-1} = 9\sqrt{-1}.$$

$$5. " \quad 5\sqrt{-1} + \sqrt{-6} = 5\sqrt{-1} + \sqrt{6 \cdot -1} \\ = 5\sqrt{-1} + \sqrt{6}\sqrt{-1}$$

6. " 0.

SUBTRACTION.

1. Ans. 0.

2. Ans. $7\sqrt{-1}$.

3. " $-7\sqrt{-1}$.

4. " $-\sqrt{-1}$.

5. " $\sqrt{-1} + 1$.

6. " $\sqrt{-1} + 3\sqrt{-3} - 16$.

MULTIPLICATION.

1. Ans. -1 .

2. Ans. $3\sqrt{-1}$.

3. " $ab\sqrt{-1}$.

4. " -6 .

5. " $-\sqrt{ab}$.

6. " $-cd\sqrt{ab}$.

DIVISION.

- | | |
|----------------------|--------------------------------|
| 1. Ans. 1. | 2. Ans. $\frac{b}{c}$. |
| 3. " $-\sqrt{-1}$. | 4. " $-\frac{a}{b}\sqrt{-1}$. |
| 5. " $-2\sqrt{-1}$. | 6. " 2. |

SECTION V.

OF LOGARITHMS.

1. Ans. The logarithm of a product, $A B$, is taken by taking the sum of the two logarithms, $\log A + \log B$.
2. Ans. The logarithm of a fraction $\frac{A}{B}$ is taken, by subtracting the logarithm of the denominator from the logarithm of the numerator; thus: $\log A - \log B$.
3. Ans. The logarithm of a power A^3 is taken by multiplying the logarithm of the basis by the exponent; thus: $3 \times \log A$.
4. Ans. The logarithm of a Root, $\sqrt[3]{A}$, is taken, by dividing the logarithm of the quantity under the radical by the radical exponent; thus: $\frac{\log A}{3}$.

ANSWERS TO THE APPLICATIONS.

- | | |
|--------------------|--------------------|
| 1. Ans. 5.5142847. | 2. Ans. 5.6758471. |
| 3. " 5.4908066. | 4. " 10.4263942. |

- | | |
|---|-----------------------------|
| 5. Ans. 0.0969100. | 6. Ans. 0.7459666. |
| 7. " 0.6690068. | 8. " 1.1972806. |
| 9. " 0.858193. | 10. " 2.5654050. |
| 11. " 2.2737376. | 12. " 0.8239087 — 1. |
| 13. " 0.7958800 — 1. | 14. " 0.0457575 — 1. |
| 15. " 0.698970 — 2. | 16. " 0.2099495 — 1. |
| 17. " 0.7106834 — 1. | 18. " 0.6480628 — 1. |
| 19. " 0.5440680. | 20. " 1.1014034. |
| 21. " 0.1324838. | 22. " 0.8450980 — 1. |
| 23. " 0.5563025 — 2. | 24. " 0.8129134 — 3. |
| 25. " 0.7510480 — 4. | 26. " 1.247857. |
| 27. " 0.8585798 — 4. | 28. " 1.381037. |
| 29. " 7.1568188.* | 30. " 18.8721901. |
| 31. " 24.0823997. | 32. " 1.162920. |
| 33. " 5.1516750. | 34. " 0.9943665. |
| 35. " 12.1667597. | 36. " 0.358635 — 3. |
| 37. " 0.142320 — 3. | 38. " 0.2518379 — 4. |
| 39. " 0.3494850. | 40. " 1.065167. |
| 41. " 0.7101112. | 42. " 0.5230012. |
| 43. " 0.9090787. | 44. " 0.0111394. |
| 45. " 0.13380. | 46. " 0.0820045. |
| 47. " 0.823908 — 1, which number corresponds to | 0.666... or $\frac{2}{3}$. |
| 48. " 0.5958482. | |

* If logarithms to 7 decimals only are employed, the above answers will not always be obtained. The last figure, therefore, may vary from the results.

ANSWERS TO THE ACTUAL CALCULATIONS OF SOME
NUMERICAL EXPRESSIONS BY MEANS OF
LOGARITHMS.

- | | |
|--------------------|--------------------|
| 1. Ans. 1.34590. | 2. Ans. 13.702 . . |
| 3. " 2.05528 . . . | 4. " 2.4855 . . . |
| 5. " 0.9593 . . . | 6. " 1.1907 . . . |
| 7. " 1.9042 . . . | 8. " 1.14605 . . . |
| 9. " 1073742500. | |

[The incorrectness of this and the following answer is occasioned by the impossibility of taking correctly the logarithms of numbers written with more than eight figures, in tables, calculated only to 5, 6, or 7 decimals ; taking therefore the corresponding numbers in the book, we must add as many cyphers on the left, as the index requires.]

- | | |
|-----------------------------|------------------------|
| 10. Ans. 2176760000. | 11. Ans. 11.8632 . . . |
| 12. " 11767.3. | 13. " 3.1681 . . . |
| 14. " 31.714 . . . | 15. " 1.443779 . . . |
| 16. " 0.0000305 . . . | |
| 17. " 0.0000000002328 . . . | |
| 18. " 0.05631 . . . | 19. Ans. 0.23256 . . . |
| 20. " 4.29980 . . . | 21. " 17.75783 . . . |
| 22. " 0.44879 . . . | 23. " 2.22818 . . . |
| 24. " 8.22508 . . . | |

SECTION VI.

ANSWERS TO THE EXERCISES IN THE
BINOMIAL THEOREM.

1. Ans. $a + b$.
2. " $a + b$.
3. " $a^2 + 2 a b + b^2$.
4. " $a^2 + 2 a b + b^2$.
5. " $a^3 + 3 a^2 b + 3 a b^2 + b^3$.
6. " $a^3 + 3 a^2 b + 3 a b^2 + b^3$.
7. " $a^4 + 4 a^3 b + 6 a^2 b^2 + 4 a b^3 + b^4$.
 $a^4 - 4 a^3 b + 6 a^2 b^2 - 4 a b^3 + b^4$.
8. " $a^5 + 5 a^4 b + 10 a^3 b^2 + 10 a^2 b^3 + 5 a b^4 + b^5$.
 $a^5 - 5 a^4 b + 10 a^3 b^2 - 10 a^2 b^3 + 5 a b^4 - b^5$.
9. " $a^6 + 6 a^5 b + 15 a^4 b^2 + 20 a^3 b^3 + 15 a^2 b^4$
 $\quad + 6 a b^5 + b^6$.
 $a^6 - 6 a^5 b + 15 a^4 b^2 - 20 a^3 b^3 + 15 a^2 b^4$
 $\quad - 6 a b^5 + b^6$.
10. " $a^7 + 7 a^6 b + 21 a^5 b^2 + 35 a^4 b^3 + 35 a^3 b^4$
 $\quad + 21 a^2 b^5 + 7 a b^6 + b^7$.
 $a^7 - 7 a^6 b + 21 a^5 b^2 - 35 a^4 b^3 + 35 a^3 b^4$
 $\quad - 21 a^2 b^5 + 7 a b^6 - b^7$.
11. " $a^8 + 8 a^7 b + 28 a^6 b^2 + 56 a^5 b^3 + 70 a^4 b^4$
 $\quad + 56 a^3 b^5 + 28 a^2 b^6 + 8 a b^7 + b^8$.
 $a^8 - 8 a^7 b + 28 a^6 b^2 - 56 a^5 b^3 + 70 a^4 b^4$
 $\quad - 56 a^3 b^5 + 28 a^2 b^6 - 8 a b^7 + b^8$.

$$12. \text{ Ans. } a^9 + 9 a^8 b + 36 a^7 b^2 + 84 a^6 b^3 + 126 a^5 b^4 \\ + 126 a^4 b^5 + 84 a^3 b^6 + 36 a^2 b^7 + 9 a b^8 \\ + b^9.$$

$$a^9 - 9 a^8 b + 36 a^7 b^2 - 84 a^6 b^3 + 126 a^5 b^4 \\ - 126 a^4 b^5 + 84 a^3 b^6 - 36 a^2 b^7 + 9 a b^8 \\ - b^9.$$

$$13. \text{ " } a^{10} + 10 a^9 b + 45 a^8 b^2 + 120 a^7 b^3 + 210 a^6 b^4 \\ + 252 a^5 b^5 + 210 a^4 b^6 + 120 a^3 b^7 + 45 a^2 b^8 \\ + 10 a b^9 + b^{10}.$$

$$a^{10} - 10 a^9 b + 45 a^8 b^2 - 120 a^7 b^3 + 210 a^6 b^4 \\ - 252 a^5 b^5 + 210 a^4 b^6 - 120 a^3 b^7 + 45 a^2 b^8 \\ - 10 a b^9 + b^{10}.$$

$$14. \text{ " } 1 + 3x + 3x^2 + x^3. \\ 1 - 3x + 3x^2 - x^3.$$

$$15. \text{ " } 1 + 4x + 6x^2 + 4x^3 + x^4. \\ 1 - 4x + 6x^2 - 4x^3 + x^4.$$

$$16. \text{ " } 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5. \\ 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5.$$

$$17. \text{ " } 32 + 240x + 720x^2 + 1080x^3 + 810x^4 + 243x^5. \\ 32 - 240x + 720x^2 - 1080x^3 + 810x^4 - 243x^5.$$

$$18. \text{ " } 625 + 2000x + 2400x^2 + 1280x^3 + 256x^4. \\ 625 - 2000x + 2400x^2 - 1280x^3 + 256x^4.$$

$$19. \text{ " } \frac{1}{16} x^4 + x^2 y + 6x^2 y^2 + 16x y^3 + 16y^4. \\ \frac{1}{16} x^4 - x^2 y + 6x^2 y^2 - 16x y^3 + 16y^4.$$

$$20. \text{ " } 243 + 810x^2 + 1080x^4 + 720x^6 + 240x^8 \\ + 32x^{10}. \\ 243 - 810x^2 + 1080x^4 - 720x^6 + 240x^8 \\ - 32x^{10}.$$

21. Ans. $9 a^2 c^2 + 12 a b c d + 4 b^3 d^2$,
 $9 a^2 c^2 - 12 a b c d + 4 b^3 d^2$.
22. " $a + 2 \sqrt{a b} + b$.
 $a - 2 \sqrt{a b} + b$.
23. " $a^2 + 6 a b + b^2 + (4 a + 4 b) \sqrt{a b}$.
 $a^2 + 6 a b + b^2 - (4 a + 4 b) \sqrt{a b}$.
24. " $a^2 b + 2 a c \sqrt{b d} + c^2 d$.
 $a^2 b - 2 a c \sqrt{b d} + c^2 d$.

ANSWERS TO THE EXERCISES IN THE APPLICATION OF THE
 GENERAL FORMULA TO THE EXTRACTION OF
 IMPERFECT ROOTS.

1. Ans. $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5 x^4}{128} \pm \&c \dots$
2. " $1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16} - \frac{5 x^4}{128} - \&c \dots$
3. " $1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5 x^3}{81} - \frac{10 x^4}{243} + \&c \dots$
4. " $1 - \frac{x}{3} - \frac{x^2}{9} - \frac{5 x^3}{81} - \frac{10 x^4}{243} + \&c \dots$
5. " $1 + \frac{x}{4} - \frac{3}{32} x^2 + \frac{7}{32} x^3 - \frac{77}{2048} x^4 \pm \&c \dots$
6. " $1 - \frac{x}{4} - \frac{3}{32} x^2 - \frac{7}{32} x^3 - \frac{77}{2048} x^4 - \&c \dots$
7. " $\sqrt{a} + \frac{x}{2 \sqrt{a}} - \frac{x^2}{8 a \sqrt{a}} + \frac{x^3}{16 a^2 \sqrt{a}}$
 $- \frac{x^4}{128 a^3 \sqrt{a}} + \&c \dots$

8. Ans. $\sqrt{a} - \frac{x}{2\sqrt{a}} - \frac{x^2}{8a\sqrt{a}} - \frac{x^3}{16a^2\sqrt{a}} - \frac{x^4}{128a^3\sqrt{a}} - \&c \dots$
9. " $a + \frac{x^2}{2a} - \frac{x^4}{8a^3} + \frac{x^6}{16a^5} - \frac{5x^8}{128a^7} \pm \&c \dots$
10. " $a - \frac{x^2}{2a} - \frac{x^4}{8a^3} - \frac{x^6}{16a^5} - \frac{5x^8}{128a^7} - \&c \dots$
11. " $a + \frac{x^3}{3a^2} - \frac{x^6}{9a^5} + \frac{5x^9}{81a^8} - \frac{10x^{12}}{243a^{10}} \pm \&c \dots$
12. " $a - \frac{x^3}{3a^2} - \frac{x^6}{9a^5} - \frac{5x^9}{81a^8} - \frac{10x^{12}}{243a^{10}} - \&c \dots$

SECTION VII.

A. *Answers to the Exercises in Arithmetical Progressions.*

1. Ans. The 14th term is 14, and the sum of all 14 terms is 105.
2. " The 13th term is 38, and the sum of all the terms is 260.
3. " The 20th term is 41, and the sum of all 20 terms is 440.
4. " The 100th term is 103, and the sum of all the terms is 5350.
5. " The 25th term is 125, and the sum of all the terms is 1625.

6. Ans. The 10th term is $11\frac{1}{2}$, and the sum of all the terms is $92\frac{1}{2}$.
7. " The 16th term is $10\frac{1}{4}$, and the sum of all the terms is 142.
8. " The 10th term is $2\frac{3}{4}$, and the sum of all the terms is $16\frac{1}{4}$.
9. " The 12th term is $2\frac{1}{8}$, and the sum of all the terms is 15.
10. " The 100th term is $35\frac{1}{2}$, and the sum of all the terms is 1775.
11. " The 26th term is $3\frac{7}{8}$, and the sum of all the terms is $60\frac{1}{8}$.
12. " The 16th term is $5\frac{2}{3}$, and the sum of all the terms is $45\frac{1}{3}$.
13. " The 14th term is $6\frac{1}{2}$, and the sum of all the terms is $45\frac{1}{2}$.
14. " The 32d term is $3\frac{9}{10}$, and the sum of all the terms is $75\frac{1}{5}$.
15. " The 36th term is $5\frac{3}{8}$, and the sum of all the terms is $114\frac{3}{4}$.
16. " The 8th term is $3\frac{1}{5}$, and the sum of all the terms is $14\frac{4}{5}$.
17. " The 100th term is 1000, and the sum of all the terms is 50500.
18. " The 50th term is $12\frac{5}{8}$, and the sum of all the terms is 325.
19. " The 22d term is $7\frac{1}{2}$, and the sum of all the terms is 88.
20. " The 10th term is $2\frac{1}{20}$, and the sum of all the terms is $16\frac{1}{2}$.
21. " The 14th term is $3\frac{2}{5}$, and the sum of all the terms is $29\frac{2}{5}$.

22. Ans. The 12th term is 10, and the sum of all the terms is 54.
23. " The 25th term is 63, and the sum of all the terms is 825.
24. " The 40th term is $8\frac{1}{2}$, and the sum of all the terms is $257\frac{1}{2}$.

B. Answers to the Exercises in Geometrical Progression.

1. Ans. The 7th term is 64, and the sum of all the terms is 127.
2. " The 10th term is 78732, and the sum of all the terms is 118096.
3. " The 9th term is 327680, and the sum of all the terms is 436905.
4. " The 10th term is 19683, and the sum of all the terms is 29524.
5. " The 12th term is 8388608, and the sum of all the terms is 11184810.
6. " The 10th term is $\frac{1}{512}$, and the sum of all the terms is $1\frac{1}{511}$.
7. " The 6th term is $\frac{1}{1024}$, and the sum of all the terms is $1\frac{1}{1023}$.
8. " The 8th term is $\frac{1}{64}$, and the sum of all the terms is $3\frac{1}{63}$.
9. " The 9th term is $\frac{1}{16384}$, and the sum of all the terms is $5\frac{1}{16383}$.
10. " The 7th term is $258\frac{2073}{4096}$, and the sum of all the terms is $591\frac{741}{4096}$.
11. " The 8th term is $106\frac{403}{512}$, and the sum of all the terms is $307\frac{1}{512}$.

12. Ans. The 6th term is $1\frac{31}{12}$, and the sum of all the terms is $19\frac{41}{12}$.
13. " The 4th term is $\frac{3}{2}$, and the sum of all the terms is $7\frac{1}{2}$.
14. " The 15th term is $\frac{1}{20\frac{1}{2}}$, and the sum of all the terms is $15\frac{2047}{18}$.
15. " The 12th term is $\frac{1}{524288}$, and the sum of all the terms is $10\frac{349325}{524288}$.
16. " The 8th term is $\frac{1458}{15625}$, and the sum of all the terms is $8\frac{9084}{46875}$.
17. " The 6th term is $15\frac{3}{16}$, and the sum of all the terms is $41\frac{9}{16}$.
18. " The 4th term is $\frac{1}{9}$, and the sum of all the terms is $4\frac{4}{9}$.
19. " The 4th term is $\frac{8}{9}$, and the sum of all the terms is $7\frac{2}{9}$.
20. " The 6th term is $\frac{1}{64}$, and the sum of all the terms is $\frac{63}{64}$.
21. " The 4th term is $\frac{1}{81}$, and the sum of all the terms is $\frac{40}{81}$.
22. " The 6th term is $\frac{1}{1096}$, and the sum of all the terms is $\frac{1365}{1096}$.
23. " The 6th term is $\frac{1}{15625}$, and the sum of all the terms is $\frac{3909}{15625}$.
24. " The 6th term is $\frac{1}{1296}$, and the sum of all the terms is $7\frac{259}{1296}$.

SECTION VIII.

Equations.

a. SIMPLE EQUATIONS.

c. ANSWERS TO THE EQUATIONS WITH ONE UNKNOWN QUANTITY.

- | | |
|------------------------------|-----------------------------|
| 1. Ans. $x = 12 - d$. | 2. Ans. $x = 5$. |
| 3. " $x = 8$. | 4. " $x = 6$. |
| 5. " $x = 5$. | 6. " $x = 4$. |
| 7. " $x = 3$. | 8. " $x = 6$. |
| 9. " $x = 7$. | 10. " $x = 3$. |
| 11. " $x = 10$. | 12. " $x = 3$. |
| 13. " $x = 6$. | 14. " $x = 4$. |
| 15. " $x = 4$. | 16. " $x = 5$. |
| 17. " $x = 5$. | 18. " $x = 11$. |
| 19. " $x = 6$. | 20. " $x = 4$. |
| 21. " $x = 5$. | 22. " $x = 8$. |
| 23. " $x = 3$. | 24. " $x = 2$. |
| 25. " $x = 2$. | 26. " $x = 4$. |
| 27. " $x = 1\frac{1}{3}$. | 28. " $x = \frac{3}{177}$. |
| 29. " $x = 60$. | 30. " $x = 48$. |
| 31. " $x = 48$. | 32. " $x = 24$. |
| 33. " $x = 30$. | 34. " $x = 9$. |
| 35. " $x = 1$. | 36. " $x = -60$. |
| 37. " $x = 1\frac{1}{3}$. | 38. " $x = 11\frac{2}{3}$. |
| 39. " $x = 139\frac{1}{2}$. | 40. " $x = 4$. |
| 41. " $x = 10\frac{2}{5}$. | 42. " $x = 66\frac{2}{3}$. |

43. Ans. $x = 288$. 44. Ans. $x = 11\frac{13}{16}$.
 45. " $x = 116\frac{143}{167}$. 46. " $x = 2571.428\dots$
 47. " $x = 10.611\dots$ 48. " $x = 2.0104\dots$
 49. " $x = 638.922\dots$ 50. " $x = -519.675\dots$
 51. " $x = \frac{b}{a}$. 52. " $x = \frac{c}{a+b}$.
 53. " $x = \frac{d-c}{a}$. 54. " $x = \frac{d-c}{a-b}$.
 55. " $x = \frac{1}{40}$. 56. " $x = \frac{1}{a}$.
 57. " $x = \frac{1}{8}$. 58. " $x = \frac{a}{b}$.
 59. " $x = \frac{3}{a+1}$. 60. " $x = 1 - \frac{2}{3}a$.
 61. " $x = \frac{a}{b+c}$. 62. " $x = 4$.
 63. " $x = \frac{c}{a+b}$. 64. " $x = \frac{c+d}{a+b+e}$.
 65. " $x = 3$. 66. " $x = \frac{a+b}{c}$.
 67. " $x = \frac{a+b+1}{cd}$. 68. " $x = \frac{3}{2^3}$.
 69. " $x = \frac{d-1}{a-b-c}$. 70. " $x = \frac{ac-3a^2bc}{c-ad}$.
 71. " $x = 16$. 72. " $x = 125$.
 73. " $x = a^2$. 74. " $x = a^3$.

b. ANSWERS TO THE EQUATIONS WITH TWO UNKNOWN QUANTITIES.

1. Ans. $x = 7$, $y = 3$. 2. Ans. $x = 10$, $y = 4$.

3. Ans. $x = 6\frac{1}{2}$, $y = 6\frac{3}{4}$.
4. " $x = 8\frac{1}{3}$, $y = 8\frac{1}{3}$.
5. " $x = 16$, $y = 35$.
6. " $x = 4$, $y = 5$.
7. " $x = 5$, $y = 4$.
8. " $x = 32$, $y = -21$.
9. " $x = 6$, $y = 4$.
10. " $x = 6\frac{12}{13}$, $y = 12\frac{7}{13}$.
11. " $x = 88\frac{3}{4}$, $y = 17\frac{3}{4}$.
12. " $x = 24$, $y = 18$.
13. " $x = \frac{1}{3}$, $y = 1\frac{1}{2}$.
14. " $x = 12.6550$, $y = 6.0750$.
15. " $x = -0.278$.. $y = -4.934$...
16. " $x = \frac{a+b}{2}$, $y = \frac{a-b}{2}$.
17. " $x = \frac{b-c}{a-d}$, $y = \frac{bd-ac}{d-a}$.
18. " $x = \frac{fc-bg}{af-bd}$, $y = \frac{dc-ag}{db-af}$.
19. " $x = \frac{bc}{a+b}$, $y = \frac{ac}{a+b}$.
20. " $x = \frac{bd}{ab+ac}$, $y = \frac{d}{b+c}$.
21. " $x = \frac{abd+ace}{b+c}$, $y = \frac{bcd-bce}{b+c}$.
22. " $x = \frac{b^2-a^2+d}{2a}$, $y = \frac{a^2-b^2+d}{2b}$.
23. " $x = 3$, $y = 7$, $z = 16$.
24. " $x = 4$, $y = 3$, $z = 2$.
25. " $x = 17$, $y = 22$, $z = 45$.

26. Ans. $x = 22\frac{2}{3}$, $y = 35\frac{2}{3}$, $z = -7\frac{1}{3}$.

27. " $x = 8$, $y = 10$, $z = 6$.

28. " $x = \frac{a + b - c}{2}$, $y = \frac{a - b + c}{2}$,
 $z = \frac{b - a + c}{2}$.

29. " $x = \frac{ce - bf}{ae - bd}$, $y = \frac{af - cd}{ae - bd}$,
 $z = \frac{a(e l - fg) - d(bl - cg)}{h(ae - bd)}$.

30. " $x = 3$, $y = 4$, $z = 5$.

31. " $x = 10$, $y = 8$, $z = 6$.

32. " $x = 24\frac{34}{635}$, $y = 7\frac{82}{635}$, $z = 19\frac{45}{635}$.

33. " $x = 13$, $y = 24$, $z = 62$.

34. " $x = \frac{2}{3}$, $y = -7$, $z = 36\frac{1}{3}$.

35. " $x = 12$, $y = 25$, $z = 6$.

36. " $x = 4$, $y = 3$, $z = 2$.

37. " $x = \frac{2}{a + b - c}$, $y = \frac{2}{a - b + c}$,
 $z = \frac{2}{b + c - a}$.

38. " This problem is undetermined; because after eliminating u , two equal equations of the forms $3x + 2y + z = 20$ are obtained. The problem therefore admits of a variety of solutions.

39. " $x = \frac{1}{4}$, $y = \frac{1}{2}$, $z = \frac{1}{4}$, $u = 0$.

40. " $x = 7$, $y = 5$, $z = 3$, $u = 1$.

B. Quadratic Equations.

1. Ans. $x = \pm 6$.

2. Ans. $x = \pm 9$.

3. Ans. $x = \pm 4$.
4. " $x = 3, x = -9$.
5. " $x = 12, x = -6$.
6. " $x = 9, x = 5$.
7. " $x = 6\frac{1}{2}, x = \frac{1}{2}$.
8. " $x = 8, x = -2\frac{1}{4}$.
9. " $x = 4, x = -13\frac{1}{2}$.
10. " $x = 8, x = -8\frac{1}{4}$.
11. " $x = 7\frac{1}{2}, x = -10\frac{1}{4}$.
12. " $x = \frac{3}{4}, x = -\frac{1}{4}$.
13. " $x = \frac{1}{3}, x = 0$.
14. " $x = \frac{2}{5}, x = 3\frac{7}{10}$.
15. " $x = \frac{1}{2}, x = -\frac{1}{3}$.
16. " $x = 6, x = -7$.
17. " $x = 5, x = -4\frac{1}{3}$.
18. " $x = 22\frac{4}{5}, x = 18\frac{2}{3}$.
19. " $x = -2\frac{1}{7}, x = 5\frac{1}{2}$.
20. " $x = 6\frac{3}{7}, x = 3\frac{1}{4}$.
21. " $x = 12, x = -5$.
22. " $x = 9, x = -6\frac{3}{8}$.
23. " $x = 3, x = 1$.
24. " $x = \frac{3}{4}, x = \frac{5}{8}$.
25. " $x = 7, x = 5\frac{3}{5}$.
26. " $x = \frac{1}{8}, x = -\frac{7}{8}$.
27. " $x = \frac{1}{5}, x = \frac{17}{15} = 1\frac{2}{15}$.
28. " $x = 6, x = x = -8$.
29. " $x = \frac{-1 \pm \sqrt{35}}{6}$; whence
 $x = 1.3699 \dots$ or $x = -1.7032 \dots$

30. Ans. $x = 0.25 \pm \sqrt{2.5625}$, whence $x = 1.8507 \dots$
or $x = -1.3507 \dots$
31. " $x = 1.3699 \dots$ or $x = -1.7032 \dots$
32. " $x = 2.5974 \dots$ or $x = -1.3474 \dots$
33. " $x = 1 + \sqrt{-9}$, or $x = 1 - \sqrt{-9}$; therefore in both cases imaginary.
34. " $x = 1 + \sqrt{-\frac{23}{2}}$, or $x = 1 - \sqrt{-\frac{23}{2}}$; therefore in both cases imaginary.
35. " $x = 6.537 \dots$ or $x = -21.253 \dots$
36. " $x = 9$, $x = 1\frac{2}{3}$.
37. " $x = 10$, $x = -\frac{2}{3}$.
38. " $x = 5\frac{2}{5}$, $x = 5$.
39. " $x = \frac{d}{c}$, $x = -\frac{b}{a}$.
40. " $x = \frac{f^2}{ag}$, $x = \frac{f^2}{ag}$.

SECTION IX.

PROBLEMS FOR SIMPLE EQUATIONS WITH ONE UNKNOWN QUANTITY.

A. Comparison of the Unknown Quantity with one or more known ones.

4. ANSWERS TO THE PROBLEMS IN WHICH THE UNKNOWN QUANTITY IS DETERMINED BY A MULTIPLE.

- | | |
|----------------|--------------|
| 1. Ans. \$100. | 2. Ans. \$8. |
| 3. " \$1. | 4. " \$2. |
| 5. " \$3. | 6. " \$5. |

7. Ans. \$2,000.
 8. " I spent \$10 and retained \$30.
 9. " \$50.
 10. " A was worth $\$789\frac{9}{18}$, but owed $\$1,578\frac{1}{8}$; B owed $\$4,736\frac{1}{8}$; and C $\$23,684\frac{4}{8}$.
 11. " C's age is $22\frac{1}{2}$ years; B's age is $45\frac{1}{2}$ years; and A's age is $91\frac{1}{2}$ years.
 12. " \$3,000.
-

b. ANSWERS TO THE PROBLEMS IN WHICH THE UNKNOWN QUANTITY IS DETERMINED BY ITS PARTS.

- | | |
|------------------------|----------------------------|
| 1. Ans. 100, Ans. 150. | 2. Ans. 25, Ans. \$12,000. |
| 3. " 60. | 4. " 48 yards. |
| 5. " \$20. | 6. " \$60. |
| 7. " \$126. | 8. " \$15. |
| 9. " \$150. | 10. " 28 men. |
| 11. " 80 years. | 12. " \$96. |
| 13. " 12 o'clock. | 14. " \$45. |
| 15. " \$24,000. | 16. " \$60. |
| 17. " 504 hds. | 18. " \$54 apples. |
-

c. ANSWERS TO THE PROBLEMS IN WHICH THE UNKNOWN QUANTITY IS DETERMINED BY BOTH ITS MULTIPLES AND PARTS.

1. Ans. The amount of the bill was \$30 : A had \$10, and B \$60.
2. " \$20.
3. Ans. 24.

4. Ans. \$2,000. 5. Ans. \$1,500.
 6. " \$1,724.13 $\frac{2}{3}$ cts. 7. " \$1,000.
 8. " A had 4 sheep, B 12, and C 3.
 9. " \$13,540. 10. Ans. \$1,750.
-

d. ANSWERS TO THE PROBLEMS IN WHICH THE UNKNOWN QUANTITY IS DETERMINED BY COMPARISON TO ITSELF.

1. Ans. \$1 $\frac{1}{2}$. 2. Ans. $\frac{1}{2}$.
 3. " A had 24, B had 12. 4. " \$9.
 5. " \$12,000. 6. " \$15,000.
 7. " \$15,000. 8. " 36,000.
 9. " The father is 60 years old, and the son is 20 years old.
 10. " B has \$4,800, and A \$6000.
 11. " There were 16 children, and 108 apples.
 12. " 31 workmen. The sum was £1, 5s. 11d.
 13. " 40 pounds.
 14. " The gentleman called for an article that was \$80 per cwt. and the merchant had two other articles, \$70 per cwt. and \$60 per cwt.
 15. " The price of the house is \$10,000. The number of his debtors is 32, and he must exact \$312.50 from each.
 16. " 5 yards.
-

e. ANSWERS TO THE PROMISCUOUS PROBLEMS BELONGING TO b, c, d, WITH SEVERAL ADDITIONAL CONDITIONS.

1. Ans. 16 lbs. 2. Ans. 10 $\frac{1}{2}$.

- | | |
|---------------------------|-------------------------------|
| 3. Ans. 12. | 4. Ans. 20 years. |
| 5. " 605 bushels | 6. " 2 miles. |
| 7. " \$20. | 8. " \$150. |
| 9. " $71\frac{1}{10}$. | 10. " A \$30, B \$40, C \$50. |
| 11. " $\$37\frac{1}{2}$. | 12. " 24. |

B. Dividing a known Quantity into two or more unknown parts.

a. ANSWERS TO THE PROBLEMS IN WHICH THE PARTS ARE IMMEDIATELY DEDUCED FROM THE WHOLE QUANTITY.

1. Ans. One receives \$500, and the other \$2,000.
2. " One receives \$15, and the other \$135.
3. " A's share is $\$5\frac{1}{3}$, B's share is \$8, C's share is $\$10\frac{2}{3}$.
4. " In 20 hours.
5. " One is to have \$250, and the other \$450.
6. " The steward is to receive \$1,000, the valet de chambre \$800, the cook \$400, and each of the 4 lackeys \$200.
7. " 8.
8. Ans. 9 of each kind.
9. " 9 days.
10. " 6 days.
11. " A must be paid for 1,800 feet, and B for 2,000 feet.
12. " 1,240 men, infantry, and the same number of cavalry.

b. ANSWERS TO THE PROBLEMS IN WHICH THE PARTS OF THE UNKNOWN QUANTITY ARE DEPENDING UPON ONE ANOTHER.

1. Ans. One has \$1,800, and the other \$3,600.

- 2 Ans. One receives \$500, and the other \$2,000.
3. " \$2,160 in notes, and \$480 in specie.
4. " 159 $\frac{2}{3}$, and 677 $\frac{1}{3}$.
5. " 26 tons of tea, 52 tons of coffee, and 182 tons of sugar.
6. " 38 students, 152 merchants, and 76 officers.
7. " 200 cavalry, 1,800 foot soldiers, and 600 artillery.
8. " 229 $\frac{2}{3}$ on horseback, 803 $\frac{1}{3}$ by water, and 2,007 $\frac{2}{3}$ on foot.
9. " 45 $\frac{1}{2}$, and 191 $\frac{1}{2}$.
10. " A's share is \$53 $\frac{1}{3}$, B's \$26 $\frac{2}{3}$, C's \$13 $\frac{1}{3}$, and D's \$6 $\frac{2}{3}$.
11. " The first receives \$40, the second \$80, and the third \$160.
12. " A's share is \$6 $\frac{2}{3}$, B's \$13 $\frac{1}{3}$, C's \$26 $\frac{2}{3}$, and D's \$53 $\frac{1}{3}$.
13. " A receives 5 guineas, B 10 guineas, C 30 guineas, and D 120 guineas.
14. " C and D's shares are \$600 each, B's share \$1,200, and A's \$2,400.
15. " B's share is \$706.20, A's is \$353.10, and C's is \$117.70.
16. " A's share is \$2,240, B's share is \$1,120, C's share is \$560, D's share is \$280.
17. " A's share is \$1,246 $\frac{4}{11}$, B's share is \$2,492 $\frac{8}{11}$, C's share is \$830 $\frac{4}{11}$.
18. " A's share is \$180, B's share is \$120, C's share is \$60.
19. " A's share is \$280, B's share is \$140, C's share is \$70, D's share is \$70.
20. " A's share is \$100, B's share is \$200, C's share is \$300, D's share is \$600, E's share is \$1200.

ANSWERS TO THE QUESTIONS BELONGING UNDER THE HEAD
OF *a* AND *b*, WITH ONE OR MORE ADDITIONAL
CONDITIONS.

1. Ans. A had \$50, and B \$49.
2. " A's share is \$24,500, and B's \$29,500.
3. " One had 112, and the other 147.
4. " A put in \$54½, B put in 45½.
5. " One is 56, and the other 40.
6. " The most needful is to receive \$68, and the other \$32.
7. " The oldest is to receive \$240, and the youngest brother \$760.
8. " One is to receive \$766⅔, and the other \$433½.
9. " The father is 69, and the son 31 years old.
10. " This problem is indefinite, and admits of an infinite number of answers.
11. " One is to receive 250 lbs, and the other 110 lbs.
12. " The first is to receive 30½ lbs, the second is to receive 34½ lbs, and the third is to receive 35½ lbs.
13. " B receives \$33½, A \$51½, and C \$15½.
14. " The youngest receives \$366⅔, the second son \$566⅔, and the third \$666⅔.
15. " The widow's share is \$4,000, each of the sons receives \$1,000, and each daughter \$500.
16. " 22 men, 18 women, and 50 children.
17. " A 2,480, B 2,204, and C 3,316.
18. " \$160.
19. " The whole sum is \$38,400. A's share is \$16,200, B's share is \$11,800, C's share is \$10,400.
20. " \$7,200.

**d. ANSWERS TO THE PROBLEMS CONTAINING THE PROGRESSIVE
DIVISION OF A KNOWN QUANTITY.**

1. Ans. 8, 9, 10, 11.
 2. " $8\frac{1}{2}$, $7\frac{1}{2}$, $6\frac{1}{2}$, $5\frac{1}{2}$, $4\frac{1}{2}$, $3\frac{1}{2}$.
 3. " \$160, \$180, \$200, \$220, \$240.
 4. " On the first day he must drink $\frac{3}{16}$ of a bottle, on the second $\frac{5}{16}$, on the third $\frac{7}{16}$, on the fourth $\frac{9}{16}$, and so on.
 5. " \$171 $\frac{3}{4}$. 6. Ans. \$2.
 7. " \$3,000. 8. " \$500.
 9. " \$4,000. 10. " by \$240.
-

**e. ANSWERS TO THE DIVISIONS OF A GIVEN NUMBER IN A
GEOMETRICAL RATIO.**

1. Ans. A's share is \$900, B's \$1,500.
 2. " $262\frac{1}{2}$, and $157\frac{1}{2}$.
 3. " A's share \$266 $\frac{2}{3}$, B's \$933 $\frac{1}{3}$.
 4. " A 135, B 297, C 432 roods.
 5. " A 144, B 240, C 210 men.
 6. " A \$270, B \$360, C \$540.
 7. " A \$3,200, B \$4,800, C \$6,000, D \$7,000.
-

**C. Comparison and Determination of numbers by
Addition and Subtraction.**

**g. ANSWERS TO THE PROBLEMS IN WHICH ONE QUANTITY
IS MADE EQUAL TO ANOTHER BY CONTINUED
ADDITION OR SUBTRACTION.**

1. Ans. 100 times.
2. Ans. 8 times 5.

- | | |
|----------------|-----------------------------|
| 3. " 45 days. | 4. " $66\frac{1}{2}$ hours. |
| 5. " 30 games. | 6. " 20 times. |

**b. ANSWERS TO THE PROBLEMS IN WHICH TWO QUANTITIES
ARE MADE EQUAL TO ONE ANOTHER BY ADDING OR
SUBTRACTING FROM BOTH.**

1. Ans. 10 days.
2. " 8 hours.
3. " $5\frac{1}{11}$ minutes, or at $5\frac{1}{11}$ minutes past 1 o'clock.
4. The equation is $200 + 40x = 90x$. Ans. 4 years.
5. The equation is $990 + 220x = 400x$. Ans. $5\frac{1}{2}$ years.
6. There are 200 bills in each pocket. The amount of the first therefore is \$400, and that of the second \$600.
7. The equation is $45 + 2x = 60 + x$; whence the Ans. 15 years.
8. The equation is $60 - 3x = 50 - 2x$; and the Ans. 10 weeks.
9. The equation is $240 + 60x = 80x$; and the Ans. 12 hours.
10. The equation is $48 + x = 2 \times (10 + x)$,
or $48 + x = 20 + 2x$;
Ans. 28 years.
11. The equation is $40 + x = 30 + 3x$. Ans. 5 years.
12. The equation is $30 + x = \frac{1}{2} \times (20 + x)$. Whence $x = 20$.
13. The equation is $20 + 2x = \frac{3}{4} \times (36 + 2x)$,
or $20 + 2x = 27 + 1\frac{1}{2}x$.
And the Ans. 14.
14. The equation is $100 + x = 80 + 2x$. Ans. \$20.

15. The equation is $60 + x = 40 + 2x$.
 Ans. 20 gentlemen with their ladies.
16. The equation is $3x + 12 = 2x + 24$.
 Ans. A had 12, and B 36.
17. The equation is $\frac{3}{4}x + 2000 = \frac{1}{2}(x + 2000)$,
 or $15x + 40000 = 16x + 32000$:
 Ans. B's fortune is \$8000, A's fortune is \$6000.
18. The equation is $x - 75 = 2(\frac{1}{5}x - 75)$,
 or $5x - 375 = 8x - 750$:
 Ans. 125, 100.

C. ANSWERS TO THE PROBLEMS IN WHICH TWO QUANTITIES
 ARE MADE EQUAL TO ONE ANOTHER, BY SUBTRACTING
 FROM THE ONE AND ADDING TO THE OTHER.

1. The equation is $7 - x = 3 + x$. Ans. 2.
2. The equation is $100 - 5x = 50 + 5x$. Ans. 5.
3. The equation is $66 - 2x = 54 + 2x$. Ans. 3 games.
4. The equation is $300 + 3x = 1000 - 4x$.
 Ans. 100 days.
5. The equation is $200000 + x = 2\frac{1}{2}(100000 - x)$,
 whence $800000 + 4x = 1100000 - 11x$.
 Ans. 20,000.
6. The equation is $8x - 400 = 5(x + 40)$. whence the
 Ans. 200 sovereigns, and 1,600 crowns.
7. The equation is $3x - 8 = 5(x - 8)$. whence
 Ans. 16 ladies and 48 gentlemen.
8. The equation is $56 + x = \frac{1}{4}(24 - x)$,
 or $224 + 4x = 264 - 11x$.
 Ans. $2\frac{2}{3}$ lbs.
9. The equation is $3x + 15 = 4(x - 15)$.
 Ans. A has \$225, and B \$75.

d. ANSWERS TO THE PROBLEMS IN WHICH ONE QUANTITY
IS MADE EQUAL TO ANOTHER BY ADDING OR
SUBTRACTING.

1. This problem is again undetermined; because the number of the garrison is not given.
2. The equation is $125x + 75(20 - x) = 2100$.
Ans. 12 men and 8 women.
3. The equation is $x + 6x + x = 72$. Ans. 9 days.
4. The equation is $\frac{x}{4} + \frac{20 - x}{6} = 3\frac{1}{2}$,
or $3x + 40 - 2x = 45$.
Ans. 5 masters and 15 journeymen.
5. The equation is $\frac{x}{4} + \frac{44 - x}{16} = 5$,
or $4x + 44 - x = 45$.
Ans. 12 quarter dollars, and 32 pences.
6. The equation is $30x = 20(8 - x) + 90$.
Ans. 5 were above 15 years old, and 3 were under 15.
7. The equation is $\frac{4}{100}x = \frac{5}{100}(1000 - x) + 4$,
or $4x = 5000 - 5x + 400$.
Ans. \$600 at 4 per cent, and \$400 at 5 per cent.
8. The equation is $75x = 25(24 - x)$,
or $3x = 24 - x$.
Ans. he worked 6 days for his master, and 18 days for other persons.
9. The equation is $70x = 20(27 - x)$,
or $7x = 2(27 - x)$.
Ans. 6 men and 21 women.
10. The equation is $6x + 5(50 - x) = 276$.
Ans. 26 and 24.

11. The equation is $4x - 3(36 - x) = 32$,
or $4x - 108 + 3x = 32$.

Ans. 20 and 16.

12. The equation is $3x = 5(40 - x)$. Ans. 25 and 15.

*D. Answers to the Promiscuous Examples for the
Exercise of the Learner.*

1. Ans. 46 years.
2. " 57 years.
3. " \$6000.
4. " Undetermined; because it is not stated what my property is.
5. " 25 lawyers, and 10 physicians.
6. " \$600. 7. Ans. \$30.
8. " 320. 9. " 35 feet.
10. They will meet in 20 hours. A will have made $\frac{2}{3}$,
and B $\frac{1}{3}$ of the whole distance.
11. x being the number of hours, the equation is
 $8x - 5x = 24$,
whence the number of hours = 8. The distance
was $64 + 40 = 104$ miles. A has come 64 miles,
and B 40 miles.
12. The equation is $x - (\frac{1}{2}x + \frac{1}{4}x + \frac{1}{8}x) = 9$,
or $12x - 6x - 3x - 2x = 108$.
Ans. 108 yards.
13. The equation is $6x - 4200 = 2400$.
Ans. \$1100 per annum.
14. The equation is $\frac{2}{3}x + \frac{1}{2}x = x + 8\frac{1}{2}$,
or $4x + 3x = 6x + 25$.
Ans. \$25.
15. The equation is $5x + 3x = 360$. Ans. 45 hours.

16. The equation is $x - (\frac{1}{10}x + \frac{1}{5}x + \frac{1}{8}x + \frac{1}{4}x) = 975$,
or $40x - 4x - 8x - 5x - 10x = 39000$.

Ans. \$3000.

17. The equation is $x - (\frac{1}{4}x + \frac{1}{5}x + \frac{1}{6}x + \frac{1}{8}x) = 930$,
or $120x - 30x - 24x - 20x = 111600$.

Ans. \$3600.

18. The equation is $\frac{2}{3}x + \frac{1}{3}x + \frac{1}{2}x + 12 = x$,
or $6x + 5x + 3x + 180 = 15x$.

Ans. The whole number of the company was 180;
the number of gentlemen = 72, the number of
ladies = 60, the number of boys = 36, and the
number of girls = 12.

19. Let the whole gain be x . Then you will have the proportions,

$22000 : 10000 :: x : \frac{22000}{10000}x = \frac{11}{5}x = \text{A's gain};$
added $22000 : 12000 :: x : \frac{22000}{12000}x = \frac{11}{6}x = \text{B's gain};$
whence, by the last condition, we have the equation
 $\frac{11}{6}x - \frac{11}{5}x = 800$. Ans. The whole gain is
\$8800. A's share is \$4000, B's share \$4800.

20. The equation is $x + 20 = 3x$. Ans. 10 years.

21. The equation is $x + x + \frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x + 2 = 150$,
or $24x + 6x + 4x + 3x = 1776$.

Ans. 48.

22. The equation is $42 + x = 5(24 - x)$,
or $42 + x = 120 - 5x$. Ans. 27.

23. The equation is $2x + 2 = 76$. Ans. 37 years.

24. The equation is $x - \frac{7}{8}x = 3$,
or $\frac{1}{8}x = 3$.

Ans. one was \$13½, and the other only \$10½.

25. The equation is $\frac{3}{4}x + 6\frac{3}{4} = x$, (because $\frac{7}{8} + \frac{3}{8}$
 $= \frac{3}{4}$); whence $\frac{9x}{40} = 6\frac{3}{4}$, or $9x = 270$ $x = \$30$.

Ans. the watchmaker values his gold watch at \$30.

26. The equation is $2x + \frac{1}{2}x - 6 = 50$. Ans. \$22 $\frac{1}{2}$, or \$22.40 cents.

27. The equation is $\frac{x}{4} + \frac{3}{8}x + \frac{x}{2} = 11\frac{1}{4}$,
or $2x + 3x + 4x = 90$. Ans. 10.

28. The equation is $\frac{x}{2} + \frac{x}{4} + 30 = x$,
or $2x + x + 120 = 4x$. Ans. 120.

29. The equation is
 $2x + \frac{1}{2}x + \frac{3}{4}x + \frac{1}{5}x + \frac{1}{10}x + 1 = 100$,
or $40x + 10x + 15x + 16x + 18x = 1980$.
Ans. 20.

30. The equation is $400 + 50x = 90x$. Ans. 10 days.

31. They shared wrong; because the 8 loaves having been divided equally among the three persons, A received $2\frac{2}{3}$ loaves; he lost therefore only $\frac{1}{3}$ loaf, while B, in receiving $2\frac{2}{3}$ loaves, lost $2\frac{1}{3} = \frac{7}{3}$ loaves. The division therefore must be made according to the loss, which is as 1 : 7. A receives 1 piece, and B receives 7 pieces.

32. The equation is $\frac{x}{3} + \frac{x}{8} + \frac{x}{10} + 318 = x$,
or $40x + 15x + 12x + 38160 = 120x$.
Ans. \$720.

33. The equation is $\frac{3}{5} \times \frac{1}{3}x = 60$,
or $\frac{1}{5}x = 60$. Ans. \$250.

34. The equation is $\frac{3}{2}x - \frac{1}{2}x = 70$,
or $9x - 2x = 420$. Ans. 60 years.

35. A's age being x , the equation is $x + x + 2\frac{2}{3}x + x + 4\frac{1}{3} = 72$; whence A's age = $21\frac{2}{3}$ years,
B's age = $24\frac{1}{3}$ years, and C's age = 26 years.

36. The equation is $15x - 12x = 27$. Ans. 9.
37. The equation is $6x - x = 45$. Ans. A has \$54, and B has \$9.
38. The equation is $\frac{1}{3}x + \frac{1}{4}x + \frac{1}{5}x + 516 = x$; whence the Ans. = \$945.
39. The equation is $19(x + 9) + 19x = 323$,
or $19x + 171 + 19x = 323$;
Ans. A travelled 22 miles, B travelled 13 miles.
40. The equation is $x + \frac{1}{2}x + \frac{1}{4}x = 280$,
or $4x + 2x + x = 1120$.
Ans. A bought 160 bushels, B bought 80 bushels, C bought 40 bushels.
41. The equation is $9\frac{3}{4}x - 12 = 300$,
or $39x - 48 = 1200$. Ans. 32 years.
42. The equation is $\frac{3x + 15}{6} + 6 = x$,
or $3x + 15 + 36 = 6x$. Ans. \$17.
43. The equation is $\frac{7x + 3}{2} - 4 = 15$,
or $7x + 3 - 8 = 30$. Ans 5.
44. The equation is $\frac{(5x - 3) \times 4 + 2}{10} = 23$,
or $20x - 12 + 2 = 230$. Ans. \$12.
45. The equation is $\frac{5x - 24}{6} + 13 = x$,
or $5x - 24 + 78 = 6x$. Ans. 54.
46. The equation is
 $\frac{1}{2}x - 100 + \frac{1}{3}x - 80 + \frac{1}{5}x - 60 + \frac{1}{4}x = x$,
or $\frac{22}{5}x + \frac{22}{5}x + \frac{11}{5}x + \frac{11}{5}x - \frac{22}{5}x = 240$.
Ans. the whole amount of his fortune is \$847 $\frac{1}{4}$.

A receives \$323 $\frac{2}{7}$, B receives \$202 $\frac{6}{7}$, C receives \$109 $\frac{7}{7}$, D receives \$211 $\frac{1}{7}$.

47. The equation is $\frac{1}{2}x - \frac{1}{4}x = 17$. The number of sheep of their friend is 204; A has 68 sheep, and B has 51.

48. The equation is $\frac{7}{8}x - \frac{3}{8}x = 400$,
or $18x - 14x = 25200$. Ans. \$4200.

49. The equation is $x + x + 2 + 2x + 6 = 96$,
or $4x = 88$.
Ans. Alexander was 24 years old, Hephæstion was 22 years old, Clytus was 50 years old.

50. The equation is $30 + x = 2(\frac{1}{2} + x)$,
or $30 + x = 1 + 2x$. Ans. in 29 years.

51. The equation is $50 + x = 2(24 - \text{ })$,
or $50 + x = 48 + 2x$. Ans. in 2 years.

52. The equation is $x + x + 150 + x + 300 = 1200$,
or $3x = 750$.
Ans. C's share is \$250, B's share is \$400, A's share is \$550.

53. The equation is $200 + \frac{1}{6}x + \frac{1}{6}x + \frac{1}{6}x = \frac{1}{2}x$,
or $1200 + 12x + 6x + 10x = 30x$.
Ans. \$600.

54. The equation is $x + 23 + x + x + 23 = 100$.
Ans. 18 years.

55. The equation is $\frac{3}{4}x - \frac{1}{2}x = 36$. Ans. They sailed 18 hours. One sailed 135 miles, and the other 99 miles.

56. The equation is $x + x + 12 + x + 28 = 400$. Ans.
A's loan is \$120, B's loan is \$132, C's loan is \$148

57. The equation is $\frac{4x}{100} \times 500 + \frac{5x}{100} \times 600 = 125$,
or $20x + 30x = 125$. Ans. $2\frac{1}{2}$ years.

58. The equation is $x + \frac{1}{3}x + 2x = 1170$. A's loan is \$270, B's is \$360, C's is \$540.

59. The equation is $x + x + 16 + x + 28 + x + 36 = 400$: A's share is \$80, B's share is \$96, C's share is \$108, D's share is \$116.

60. The equation is $9x - 18 = 48$. The first receives \$7 $\frac{2}{3}$, the second \$6 $\frac{1}{3}$, the third \$6 $\frac{2}{3}$, the fourth \$5 $\frac{1}{3}$, the fifth 5 $\frac{2}{3}$, &c.

61. The equation is formed in the following manner:

$$\frac{1}{3}x - 1000 = \text{end of the first year};$$

$$\left. \begin{aligned} \frac{1}{3}(\frac{1}{3}x - 1000) - 1000 \\ = \frac{1}{9}x - 4000 - 3000 \\ = \frac{1}{9}x - 7000 \end{aligned} \right\} \text{end of the 2d year};$$

$$\left. \begin{aligned} \frac{1}{3}(\frac{1}{9}x - 7000) - 1000 \\ = \frac{1}{27}x - 23000 - 3000 \\ = \frac{1}{27}x - 26000 \end{aligned} \right\} \text{end of the 3d year}.$$

$$\text{Consequently, } \frac{1}{27}x - 26000 = 2x,$$

$$\text{or } 64x - 111000 = 54x. \text{ Ans. } \$11,100.$$

62. The equation is $50x + 75x = 500$. Ans. 4 days.

63. The equation is $8x + 7(20 - x) = 144$,
or $8x + 140 - 7x = 144$.

The men were 4 in number, and the women 16.

64. The equation is $4\frac{1}{3}x + 2\frac{1}{2} = 5x - 10$,
or $39x + 20 = 40x - 80$.

Ans. 100 lbs.

65. The equation is
 $x + 4x - 300 + 12x - 1100 + 24x - 2300 = 8600$,
or $41x = 12300$.

D's share is \$300, C's share is \$900, B's share is \$2500, A's share is \$4900.

66. The equation is formed in the following manner :

$$\frac{4}{5}(x - 10) - 2 = \text{what he breaks,}$$

$$x - 10 - [\frac{4}{5}(x - 10) - 2] =$$

$$x - 10 - (\frac{4}{5}x - 8 - 2) =$$

$$x - 10 - \frac{4}{5}x + 8 + 2 = \frac{1}{5}x = \text{what he has left. Consequently } \frac{1}{5}x + 53 = x - 11. \text{ Ans. 80 eggs.}$$

67. The equation is $30 + 6x = 8x$. Ans. 15 days.

68. The equation is formed in the following manner :

$$2x - 2 = \text{what she had after leaving Jupiter's temple ;}$$

$$4x - 4 - 2 = 4x - 6 = \text{what she had after leaving the temple of Apollo ;}$$

$$\text{whence } 4x - 6 = 2x. \text{ Ans. She had 3 drachms at first.}$$

69. The equation is $3\frac{1}{2}x + 5\frac{1}{6}x = 80 - 28 = 52$,

$$\text{or } 21x + 31x = 312. \text{ Ans. 6 hours.}$$

70. The equation is $\frac{3}{4}x - \frac{3}{8}x = 8$,

$$\text{or } 24x - 21x = 448. \text{ Ans. } 149\frac{1}{3} \text{ yards.}$$

71. The equation is $x + \frac{25}{100}x = 2000$. Ans. \$1600.

72. The equation is $x + \frac{80}{100}x = 900$. Ans. \$500.

73. The equation is $4(x - 1) = 3x + 6$. There were 10 soldiers, and the amount of money \$36.

74. The equation is $90 + \frac{15}{4}x = \frac{20}{3}x$,

$$\text{or } 1080 = 80x - 75x. \text{ Ans. 216 hours.}$$

75. The equation is $6x + 3x + 75 = 225$. Ans. \$16 $\frac{2}{3}$ is the price of the new wine per hhd, and 41 $\frac{2}{3}$ of the old wine.

76. The equation is $800x + 700x = 6000$. Ans. In 4 minutes.

77. The equation is $800x = 6000 + 700x$. Ans. in 60 minutes, or 1 hour.

78. The equation is $61 + x = 3(13 + x)$. Ans. In 11 years.

79. The equation is formed in the following manner :

$$200 + \frac{x-200}{2} = \frac{1}{2}x + 100, = \text{the valet de chambre's;}$$

$$\frac{x - \frac{1}{2}x - 100}{5} + 400 = \frac{x + 3800}{10} = \text{the cook's;}$$

$$\text{consequently } \frac{1}{2}x + 100 + \frac{x + 3800}{10} + 500 = x.$$

Ans. \$2450.

80. The equation is $15x = 10x + 80$. Ans 16 yards of the first and 24 yards of the second.

81. The equation may be formed as follows :

$$\frac{3}{2}x + 1 = \text{end of the first game;}$$

$$\frac{3}{2}(\frac{3}{2}x + 1) + 2 = \frac{3}{4}x + \frac{3}{2} + 2 = \frac{9x + 14}{4};$$

= end of 2d game ;

$$\text{and consequently } \frac{3}{2}\left(\frac{9x + 14}{4}\right) + 3 = 4x,$$

$$\text{or, } 27x + 42 + 24 = 32x.$$

Ans. \$13\frac{1}{2}.

82. The equation is $12x - 18 = 9x + 12$. Ans. She wishes to buy 10 yards, and has 8s. 6d.

83. The equation is $\frac{1}{4}x - \frac{7}{5}x = 100$. Ans. $583\frac{1}{5}$ yards.

84. The equation is $70 + \frac{4}{100}x \times 500 = \frac{4}{100}x \times 480$,
or $70 + 20x = 24x$. Ans. in $17\frac{1}{2}$ years.

85. Ans. \$108.

86. The equation is $x + 2x + 1 + 4x + 3 = 102$.

The first was 14 years old, the second was 29 years old, the third was 59 years old.

87. The equation is $18x + 15x + 30 = 228$. Ans. The sugar is 6*d* per lb, and the coffee 8*d*.

88. The equation is $x + 28 = 5x$. He was 35 years, and his daughter 7 years old.

89. The equation is formed as follows :

$$2x - 800 = \text{end of 1st year ;}$$

$$4x - 1600 - 1600 = 4x - 3200 = \text{end of 2d year ;}$$

$$\text{and consequently } 8x - 6400 - 2400 = 0. \text{ Ans. } \$1100.$$

90. The equation is $200x - 700 = 150x + 1100$.
Ans. \$6500.

91. The equation is $2x + 1\frac{1}{2}(120 - x) = 1\frac{3}{4} \times 120$,
or $2x + 180 - \frac{3}{2}x = 200$.
Ans. 40 bottles at \$2, and 80 bottles at \$1.50.

92. Ans. 8 days.

93. The equation is $480 + 40x = \frac{8}{3} \times 40x$,
or $1440 + 120x = 320x$. Ans. $7\frac{1}{2}$ days.

94. The equation is $\frac{12}{100}x + \frac{10}{100}(10000 - x) = 1080$,
or $12x + 10000 - 10x = 108000$.
Ans. In the one he has \$4000, and in the other \$6000.

95. x = number of gallons of first cask, $\frac{1}{3}x$ = number of gallons of 2d cask, $\frac{4}{5}x$ = number of gallons of 3d cask ; consequently, $x = \frac{1}{3}x + \frac{4}{5}x + 30$. Ans. the first cask contains 135, the 2d cask contains 45, and the third 60 gallons.

96. This problem is similar to problem 31. Ans. A must pay 10*s*. and B 1*s*.

97. The equation is $1.25x - 7.50 = 0.90x + 13.50$.

Ans. 60 lbs. is the weight, and the prime cost \$1.12½ cents per lb.

98. The equation is $5x - 16 = 2(x + 24)$.

Ans. B had \$21½, and A had \$106¾.

99. The equation is $\frac{7x}{100} - \frac{1}{100}(24000 - x) = 600$,

$$\text{or } 7x - 120000 + 5x = 60000.$$

Ans. 15000 at 7 per cent, and \$9000 at 5 per cent.

100. The equation is $\frac{5}{2}x - x = 3000$. Ans. The one 2000 feet, and the other 5000 feet.

101. The equation is $\frac{x}{7} + \frac{46 - x}{3} = 10$. Ans. 28, 18.

102. The equation is formed thus :

$$x = \text{first,} \quad 2x + 1 = 2d,$$

$$6x + 6 = 3d; \text{ hence } 9x + 7 = 70.$$

Ans. 7, 15, 48.

103. The equation may be formed thus :

Let x = the hire of 1 man in pence,

then $\frac{328 - 12x}{8}$ the hire of one woman,

(328 pence being the whole pay).

Again ; from condition 2d,

$$\frac{370 - 12x}{11} = \text{hire of 1 woman.}$$

$$\text{consequently } \frac{328 - 12x}{8} = \frac{370 - 12x}{11};$$

whence 18d. = the hire of a man, 14d. = the hire of a woman.

104. This problem is similar to problem 91.

Ans. 66¾ gallons at \$1.60, and 133½ gallons at \$1.00.

105. Let x = the whole fortune ;

$$100 + \frac{x-100}{10} = 90 + \frac{1}{10}x = \text{share of 1st,}$$

$$200 + \frac{x-290-\frac{1}{10}x}{10} = 171 + \frac{9}{100}x = \text{share of 2d.}$$

and because they were to share equally,

$$90 + \frac{1}{10}x = 171 + \frac{9}{100}x;$$

$$\text{whence } 9000 + 10x = 17100 + 9x.$$

Ans. His whole fortune was \$8100. The share of a child \$900, and the number of children 9.

106. Ans. 1001 cubic feet from the larger, and 440 from the smaller.

107. The equation is $50 + x = \frac{1}{2}x$. Ans. 700 paces.

108. The equation is $27 + \frac{3}{4}x = x$. Ans. 189 throws.

109. Ans. A must receive £5, B only 10s. and C nothing; because in either case he shares $\frac{1}{2}$ of the gain or loss.

110. The equation is $1200 = 500 + \frac{1}{2}B$.

whence B's share = 280, A's share = 720, C's share = 840, D's share = 360.

111. The equation is $2x + 24 = 3x$.

The first is worth \$24, and the 2d \$64.

112. The equation is $\frac{4x + 400}{5} = \frac{1}{2}x + 40$. Ans. \$700.

113. Ans. $\frac{1}{3}$.

114. The equation is $\frac{5}{8} + \frac{5}{8} \times \frac{1}{x} = \frac{1}{2}$,

$$\text{or } \frac{5}{6x} = \frac{1}{2} - \frac{5}{8}.$$

Ans. by $\frac{1}{3\frac{1}{5}}$ part.

115. The equation is $2x - 2 = x + 1$.

Ans. 3 girls and 4 boys.

116. Let $x =$ the cover,
 $6 + \frac{1}{2}x =$ the 2d cup, whence
 $6 + \frac{3}{2}x = 36$.
 Ans. The cover weighs 20 ounces, and the 2d cup
 16 ounces.
117. Ans. 5600 feet.
118. The equation is $3x + 3850 - 5x = 2350$.
 A has \$250, B has \$320.
119. The equation is $9x + 250 - 5x = 400$.
 Ans. $37\frac{1}{2}$ gallons at 9s., $12\frac{1}{2}$ gallons at 5s.
120. This problem is similar to many preceding ones.
 Ans. A lost \$2, B lost \$6 $\frac{1}{2}$, C lost \$11, D lost \$8 $\frac{1}{4}$,
 E lost \$12 $\frac{1}{4}$.
121. Ans. $13\frac{1}{3}$ oz. of 14 carets fine, $6\frac{2}{3}$ oz. of 8 carets fine.
122. Ans. $6\frac{2}{3}$ bottles.
123. The equation is $420 + 12x = 325$. Ans. $8\frac{1}{4}$.
124. Ans. 30 lbs.
125. " 8 fourpences, and 9 quarter dollar pieces.
126. " 10 lbs.
127. " $1\frac{1}{3}$ hours.
128. " 48 minutes.
129. The equation is $\frac{2}{3}x + \frac{2}{4}x + \frac{2}{5}x = 756$.
 Ans. $137\frac{13}{31}$ days.
130. Ans. 20 cubic inches.
131. Ans. The company consisted of 100 persons; the
 sum which was to be collected was \$50, and the
 contribution of each 50 cents.
132. The watch costs \$120, and he sold 80 tickets.
133. The equation is $x - 6 = 4 \left(\frac{x}{3} - 6 \right)$,

$$\text{or } x - 6 = \frac{4x}{3} - 24.$$

The father is 54 years old, and the son is 18 years old.

134. Ans. \$240.

135. The equation is $\frac{5}{8}x = \frac{4}{5}(9800 - x)$.

Ans. One had \$4800, and the other had \$5000.

136. The equation is $70x - 70 = 70 - 28x$,
whence $x = 1\frac{2}{3}$ = number of sheets written in
one hour, and as he works 28 hours in a week, he
will write $28 \times 1\frac{2}{3} = 40$ sheets.

137. The equation is $\frac{27}{4} - \frac{1}{2}x + x = 29$.

A has \$19 $\frac{1}{2}$, and B \$14 $\frac{1}{2}$.

138. The equation is $4x + 4x + 2\frac{1}{2}x = 52\frac{1}{2}$.

Ans. 5 yards of the best kind, 10 yards of the
second kind, 20 yards of the third kind.

139. The equation is $x + \frac{5}{6}x + \frac{3}{2}x + 2x = 1170$,

or $10x + 12x + 15x + 30x = 11700$;

whence D's gain \$174 $\frac{2}{3}$, C's gain \$209 $\frac{2}{3}$, B's
gain \$261 $\frac{2}{3}$, A's gain \$349 $\frac{1}{3}$.

140. Ans. 360 feet.

141. The equation is $5x - 20 = 20 - x$. Ans. 6 $\frac{2}{3}$.

142. The equation is $\frac{1400 + 16x}{56} = \frac{1725 + 30x}{84}$,

or $4200 + 48x = 3450 + 60x$.

Ans. 62 $\frac{1}{2}$ cents.

143. The equation is $\frac{3}{4}(\frac{3}{2}x + 50) + 70 = 120$,

or $\frac{1}{2}x + 37\frac{1}{2} + 70 = 120$. Ans. \$25.

144. Ans. \$275.

145. This problem is solved in the same manner as the
preceding, (144). Ans. 80.

146. Ans. $9\frac{1}{2}$ months.
147. " $2\frac{1}{3}$ months.
148. " The first instalment is due in $7\frac{1}{2}$ months, the second $7\frac{1}{2}$ months after that, and so on.
149. " \$7936.
150. " In $3\frac{3}{4}$ months.
151. " \$7722.
152. " A receives \$208 $\frac{1}{3}$, B 108 $\frac{1}{3}$, C 182 $\frac{1}{3}$.
153. " A contributed \$2450, B \$3675, C \$6425.
154. " A \$3450, B \$3770.
155. " The rate of interest is 8 per cent. per annum, or $\frac{2}{3}$ per cent. per month; and it would take 6 children 10 months to spend at the same rate \$1650.
156. " \$110 $\frac{2}{3}$.
157. " 40 eggs.
158. " 31.
159. This problem has already occurred before.
Ans. \$11000.
160. We have the proportion $\frac{1}{4}^3 : \frac{1}{4} : 1 : x$. Ans. $\frac{1}{13}$ of an hour.
161. The equation may be formed in the following manner:
 $\frac{5}{6}x - 1000$ at the end of the first year,
 $\frac{2}{3}\frac{5}{6}x - 2200$ at the end of the 2d year,
 $\frac{1}{2}\frac{2}{3}\frac{5}{6}x - 3640$ at the end of the 3d year,
 which must be equal to $\frac{5}{6}x + 200$. Ans. \$30000.
162. The equation is $(3\frac{3}{4}x - 60)2\frac{1}{2} - 30 = 0$, whence the number is = 21.
163. Ans. 1975 men.

164. Ans. \$355.

165. The equation is $\frac{1}{12}x \times \frac{1}{12}x = x$,
or $\frac{1}{144}x = 1$. Ans. 144 oxen.

166. The equation is $\frac{7}{12}x + 50 = x$. Ans. The third cask contains 120 gallons, the 2d cask contains 90 gallons and the 1st cask contains 70 gallons.

167. Ans. The first contains 140 gallons, the 2d contains 60 gallons, the 3d contains 45 gallons, the 4th contains 80 gallons.

SECTION X.

Answers to the Questions in Simple Equations with two or more unknown Quantities.

1. Ans. 40 and 30.

2. " The first \$180, the second \$120.

3. " One contains \$20, and the other \$30.

4. The two equations are $x + 100 = y - 100$,
 $y + 100 = 2x - 100$.

Ans. A has \$500, B \$700.

5. Ans. The first is worth \$24, the second \$64.

6. The two equations are $x + y = 570$,
 $3x + 5y = 2350$.

Ans. A has \$250, B \$320.

7. The two equations are $x + 2 = y - 2$,
 $y + 2 = 2x - 4$.

Ans. One had 10, and the other 14.

8. The equations are $2x + 5y = 31$,
 $7x + 4y = 68$. Ans. 8 and 3.

9. The equation is $x + 4 = 3\frac{1}{2}y$,
 $y + 8 = \frac{1}{2}x$. Ans. 48 and 16.

10. The equations are $x - 6 = 3\frac{1}{3}y$,
 $x + 3 = 2\frac{1}{6}y$.
 Ans. The father 36, and the son 15 years.

11. The equations are $x + y = 9800$,
 $\frac{5}{8}x = \frac{4}{5}y$.

Ans. A has \$4800, and B \$5000.

Remark. The same sum has been solved before, with *one* unknown quantity.

12. The equations are $\frac{3}{4}y + x = 600$,
 $\frac{1}{2}x + y = 600$.

Ans. A has \$240, and B \$480.

13. The equations are $x + \frac{1}{8}y = 1200$,
 $y + \frac{1}{6}x = 2500$.

Ans. A had \$906 $\frac{1}{3}$, B \$2348 $\frac{1}{3}$.

14. The two equations are $\frac{1}{4}x = \frac{1}{5}y$,
 $\frac{1}{4}x + \frac{1}{5}y = 8$.

Ans. A had \$16, and B \$20.

15. The equations are $\frac{2}{3}x = \frac{3}{4}y$,
 $\frac{1}{3}x + \frac{1}{4}y = 10$.

Ans. One had \$18, and the other \$16.

16. Ans. At 4 $\frac{1}{2}$ and 5 $\frac{1}{2}$ per cent.

17. The two equations are $\frac{3}{4}y - 96 = \frac{2}{3}x$,
 $\frac{5}{8}y = \frac{4}{5}x$.

The first weighs 720 lbs, the 2d 512 lbs.

18. Ans. The first pipe discharges 15, and the second 6 buckets; both will require 10 hours to fill the cistern.

19. The two equations are $x + y = 500$,
 $20x + y = 1694$.
 Ans. 326 sovereigns, 174 shillings.
20. The two equations are $4x + 4y = 28$,
 $4x + 5y = 33$.
 The price of an orange was 2 cents, and that of a lemon 5 cents apiece.
21. The two equations are $60x + 40y = 3000$,
 $3x = 4y$.
 The price of the coffee was $33\frac{1}{3}$ cents per lb, and that of the sugar 25 cents per lb.
22. The two equations are $8x + 9y = 1846$,
 $20x + 16y = 3640$. Ans. 62 cents and \$1.50.
23. The two equations are $15x + 33y = 39\frac{1}{2}$,
 $24x + 55y = 65$.
 Ans. The Silisian ell is to the Brabant ell as 5 to 6. The Leipzig to the Brabant as 9 to 11, the Silisian to the Leipzig as 55 to 54.
24. The French mile is to the German as 3 to 5; the English to the German as 23 to 106; and the French to the English as 318 to 115.
25. The two equations are $x + 50 - 8 = y + 2$,
 $3\frac{1}{2}(x + 2) = y + 50$.
 Ans. The first horse is worth \$30, and the 2d \$70.
26. Ans. $\frac{1}{5}$.
27. The two equations are $\frac{x-3}{y-3} = \frac{1}{4}$,
 $\frac{x+5}{y+5} = \frac{1}{2}$. Ans. $\frac{7}{5}$.
28. Ans. A has lent \$10000, B \$22600, C \$13000; A at 4 per cent, B at 5, and C at 6 per cent.

29. Ans. There were 11 persons in the company, and each spent 80 cents.

30. The second equation is $(x - 2)(y - 3) = xy - 145$.

Ans. 29 lines per page, and 32 letters in a line.

31. Ans. The wheat is \$5, and the rye \$2.75 per barrel.

32. " The first holds 22, and the second 10 gallons.

33. " The best wine is \$1.12 per gallon, and the worst 80 cents.

34. " $\frac{2}{3}$.

35. The two equations are $x + y = 120$,

$$\frac{5}{37}x + \frac{2}{23}y = 14.$$

Ans. 74 lbs. of tin, and 46 of lead.

36. Ans. 112 lbs of silver, and 36 lbs of copper.

37. " 14.77 . . of gold and 5.22 . . . of silver.

38. " 2 and 10.

39. " 5 and 8.

40. " 3 and 6.

41. " 4 and 16.

42. The three equations are $x + y = 54$,

$$y + z = 109,$$

$$x + z = 85.$$

Ans. He himself is 18 years old, his father is 38 years old, and is grandfather 62.

43. The two equations are $x - 7 = 3y - 21$,

$$x + 7 = 2y + 14.$$

Ans. The age of the father is 49, and that of his son 21.

44. The equations are $x + \frac{2}{3}y = 2190$,

$$y + \frac{1}{3}z = 2190,$$

$$\frac{2}{3}x + z = 2190.$$

Ans. A \$1530, B \$1540, C \$1170.

45. Ans. A has \$200, B \$360, C \$840.
46. " In the first were \$120, in the second \$380, and in the third \$500.
47. " A \$980, B \$1540, C \$2380.
48. " 20, 28, and 50.
49. " A \$400, B \$640, C \$780.
50. " They spent \$6½; A has \$5, and B \$6.
51. " The 15 carats fine weighs 8 lbs, the 10 carats 16, and the 9 carats weighs 10 lbs.
52. " The barrel of wheat is worth \$7, the barrel of rye \$6, and the barrel of barley \$4.
53. " The coffee 75 cents, the sugar 50 cents, and the tea \$2.00.
54. " A \$52, B \$28, C \$16.
55. " 37, 25, 21.
56. " In the first there were \$70, in the 2d \$52, and in the 3d \$40.
57. " 30, 48, 50.

SECTION XI.

PROBLEMS WHICH LEAD TO QUADRATIC EQUATIONS.

A. Answers to the Problems which lead to pure Quadratics.

- | | |
|-------------|-------------|
| 1. Ans. 12. | 2. Ans. 14. |
| 3. " 24. | 4. " 72. |
| 5. " 120. | 6. " 224. |

7. Ans. 18. 8. Ans. 50, and 15.
 9. " 85 and 76. 10. " 12 persons.
 11. " Undetermined; because it is not stated what he paid for them.
 12. " \$6480.
 13. Undetermined.
 14. The equation is $2\frac{1}{2} \times \frac{x}{1000} = x$,
 or $\frac{5x}{2000} = 1$. Ans. 20 merchants.
 15. Ans. \$2631 66 cents, nearly.
 16. " Of the first 15, of the 2d 20, and of the 3d 70 lbs.
 17. The equation is $\frac{1}{5}x^2 - 125 = 125 - \frac{x^2}{2}$,
 or $26x^2 - 1250 = 1250 - 5x^2$,
 or $31x^2 = 2500$.
 Ans. 8.98... lbs.
 18. The equation is $\frac{4}{3}x^2 = 2352$. Ans. 42.

B. Answers to the Problems leading to Mixed Quadratic Equations.

1. The equation is $x^2 + 8x = 240$. Ans. 12 and 20.
 2. The equation is $x^2 + 59x = 1200$. Ans. 16 persons.
 3. The equation is $x^2 + 6x = 91$. Ans. 7 broad and 13 long.
 4. The equation is $x^2 - x = 306$. Ans. 18.
 5. Ans. 48.
 6. The equation is $x^2 + 6x = 135$. Ans. He paid \$9 a day, and stayed 15 days.
 7. Ans. 42.

8. The equation is $x^2 = 36x + 832$. Ans. 16 oxen.
9. The equation is $\frac{x^2}{100} + x = 119$. Ans. \$120.
10. Ans. \$40, \$72, and \$80.
11. " 12 pieces.
12. " 54.
13. The equation is $x^2 + 10x - 2475 = 900000$.
Ans. 945.
14. The equation is $3x^2 + 6x + 5 = 245$.
Ans. The first worked 8 days, and received 8s. per day, the 2d worked 9 days, and received 9s. per day, and the 3d worked 10 days, at 10s. per day.
15. The equation may also be $\frac{1200}{x} + 5 = \frac{1200}{x - 40}$,
or $x^2 - 40x = 9600$.
The first had 120 men, and the 2d had 80. Each soldier in the first company received \$10; and each soldier in the 2d company received \$15.
16. The equation is $4x^2 + 4x = 1680$. Ans. 20 roods broad, and 84 rods long.
17. The equation is $x^2 - 11x + 30 = 3x$,
or $x^2 - 14x = -30$.
Ans. \$11.358 ... or \$2.642 ...
18. Ans. 69.53 ... lbs.
19. " He is 35, and his brother 36 years old.
20. The equation is $\frac{24}{x} = \frac{24}{20 - x} + 1$. Ans. 8 men, and 12 women.
21. Ans. \$80.
22. The equation is $\frac{100}{x + 7} + \frac{100}{x} = 43\frac{1}{3}$. Ans. 3 and 10.

23. Ans. \$300.

24. " \$20.

25. The equation is $\frac{9x}{x+30} = \frac{4x+120}{x}$.

Ans. The distance between C and D is 150 miles.
B travelled 60 miles, and C 100 miles.

26. Ans. 15 and 16.

27. Ans. 12 and 20.

28. " 15 and 17.

29. " Into 4 and 12.

30. " 1296.

31. " $56\frac{1}{4}$.

32. " One \$200, and the other \$300.

33. " One \$1200, and the other \$800.

34. " 9 and 15.

35. Ans. 862.

36. " 654.

37. " 6. 24, 96.

APPENDIX.

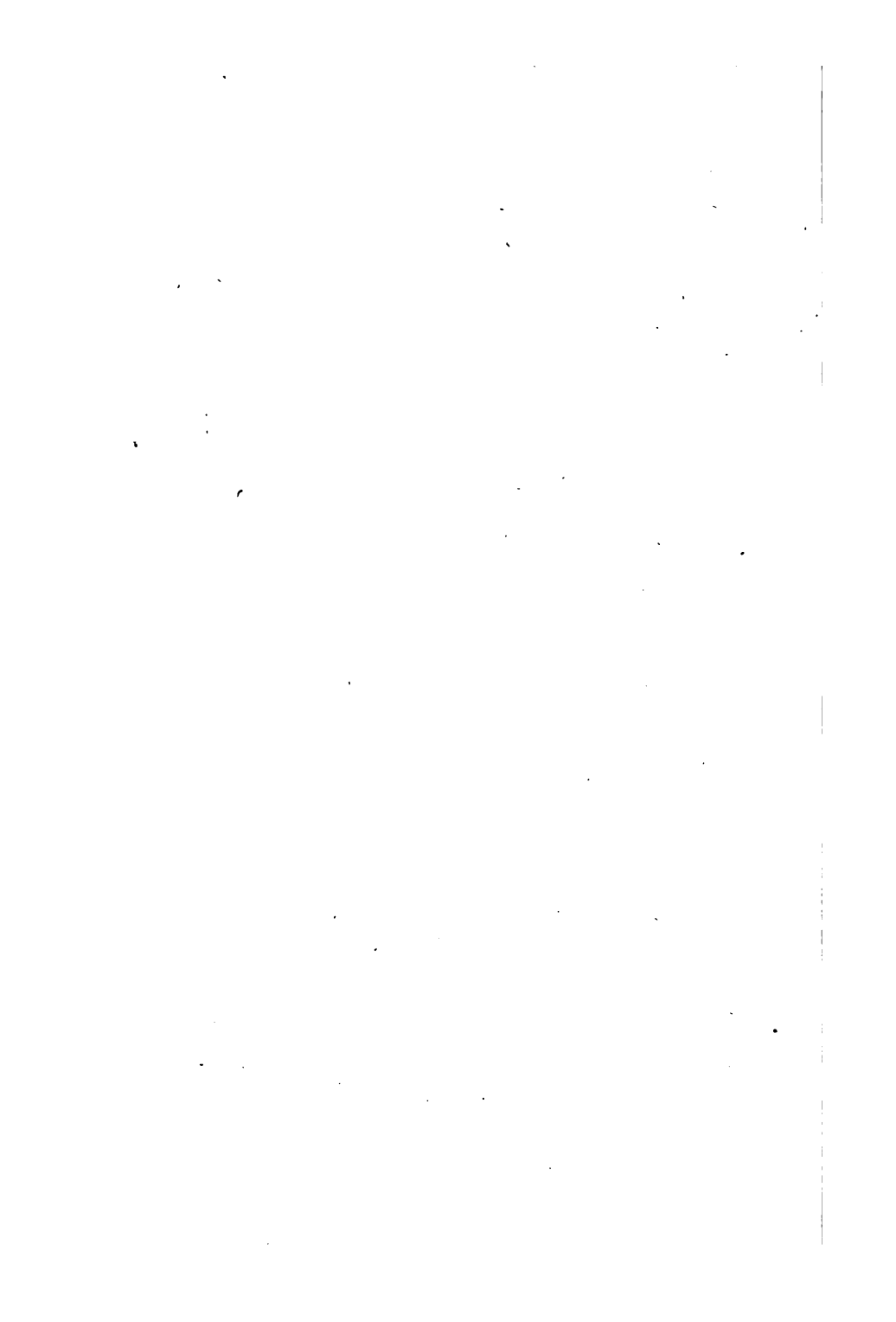
CONTAINING PROBLEMS IN COMPOUND INTEREST AND ANNUITIES, FOR THE USE OF LOGARITHMS.

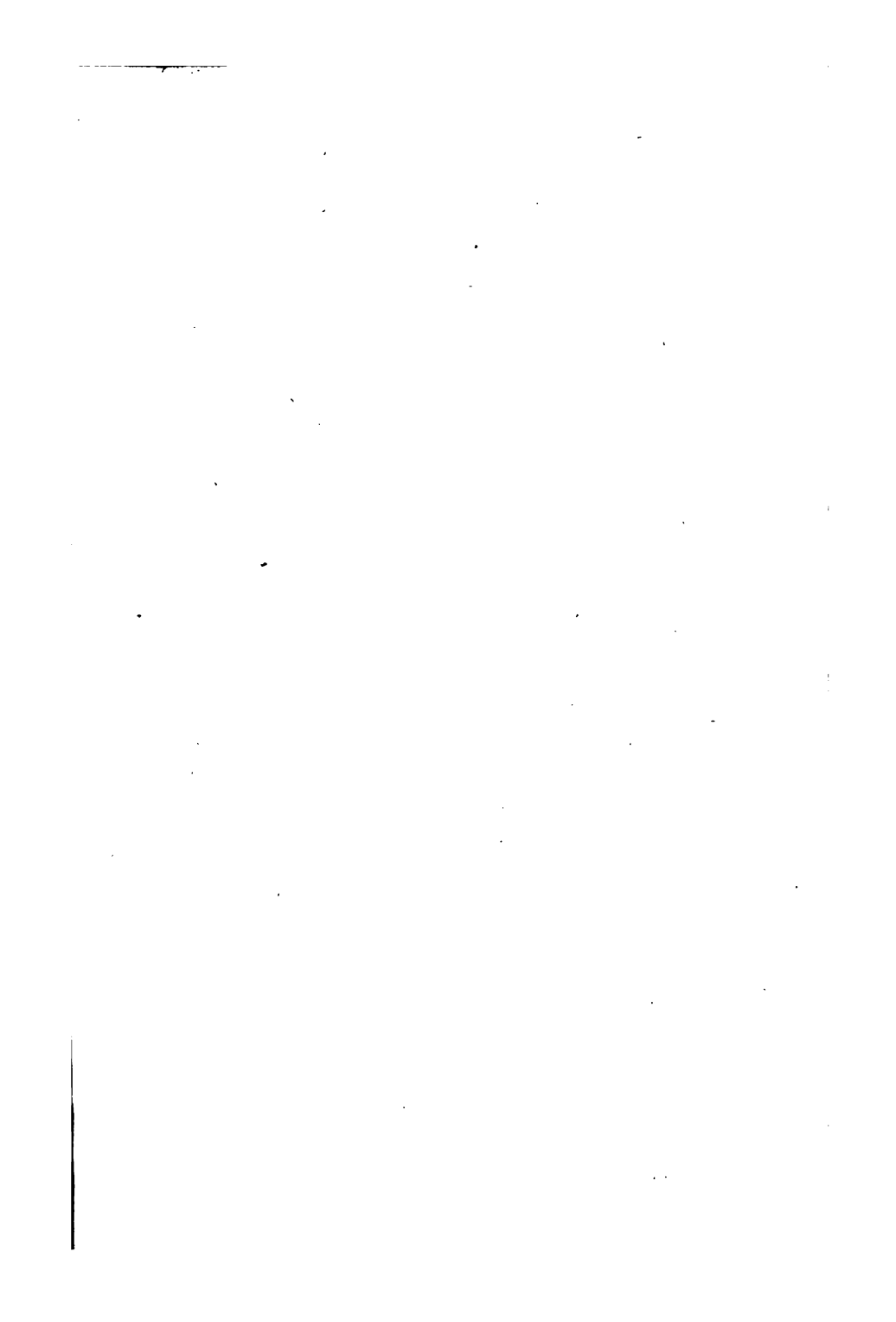
ANSWERS TO THE APPLICATIONS OF THE FORMULAS.

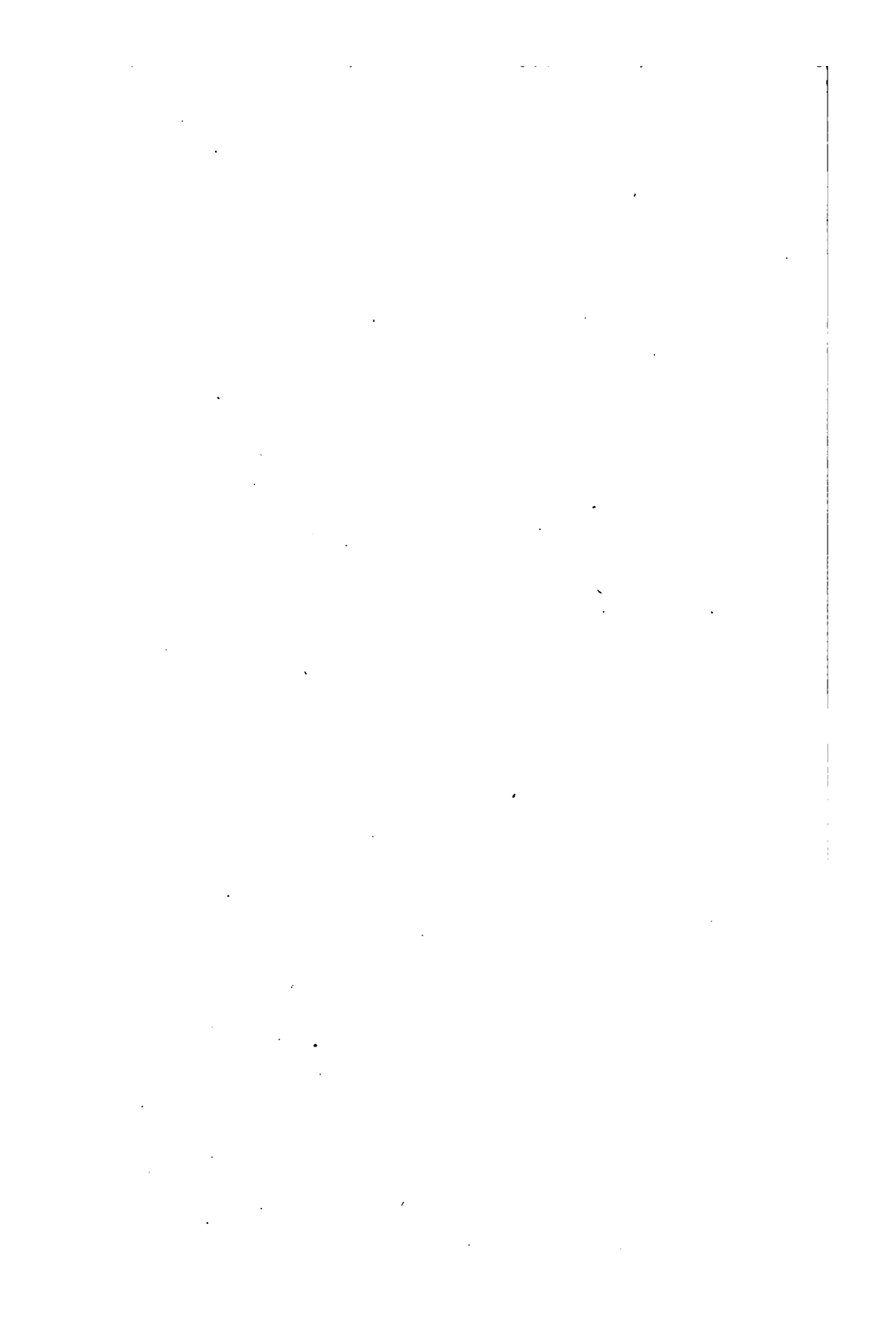
- | | | | |
|-------------------|---------|-----------------------------|---------|
| 1. Ans. \$1343.90 | nearly. | 17. Ans. \$5128.80 | nearly. |
| 2. " \$2653.36 | " | 18. " \$771.89 | cents. |
| 3. " \$7401.30 | " | 19. " \$644.61 | nearly. |
| 4. " \$10955.45 | " | 20. " \$6755.65 | " |
| 5. " \$24005.00 | " | 21. " \$310.86 | " |
| 6. " \$34050.84 | | 22. " \$12424.20 | cents. |
| 7. " \$3385.55 | nearly | 23. " \$1884.44 | cents. |
| 8. " 14489276 | " | 24. " \$27919.80 | nearly. |
| 9. " 6890190000 | " | 25. " \$6246 | " |
| 10. " \$35917.10 | " | 26. " is given in the book. | |
| 11. " \$49744.60 | " | 27. " a little over 3 per | |
| 12. " \$24924.12 | " | | cent. |
| 13. " \$21673.30 | " | 28. " \$3683.48 | cents. |
| 14. " \$148.59 | " | 29. " \$3350.37 | nearly. |
| 15. " \$136.72 | " | 30. " \$3322 | " |
| 16. " \$22.08 | | 31. " \$228 | |

- | | |
|--|---|
| 32. Ans. a little over $13\frac{3}{4}$ per cent. | 48. Ans. in between 23 and 24 years. |
| 33. " a little over 18 per cent. | 49. " in a little over 4 years. |
| 34. " nearly $28\frac{1}{2}$ per ct. | 50. " nearly 27 years. |
| 35. " a little over 31 per cent. | 51. " in a little over 22 years, at 5 per ct. |
| 36. " by a little more than $4\frac{1}{2}$ per ct. | and in about 19 years, at 6 per ct. |
| 37. " little over 2 per ct. | 52. " in between 13 and 14 years. |
| 38. " " " $1\frac{3}{8}$ " | 53. " in a little over 20 years. |
| 39. " " " 3 " | 54. " in about 88 years. |
| 40. " a little more than 247 millions. | 55. " nearly 4 per cent. |
| 41. " is given in the book. | 56. " a little over $4\frac{3}{4}$ per cent. |
| 42. " between 11 and 12 years. | 57. " between 16 and 17 years. |
| 43. " is between 13 and 14 years. | 58. " a little over 9 years. |
| 44. " between 40 and 41 years. | 59. " \$3621.29 nearly. |
| 45. " between 17 and 18 years. | 60. " \$20429.50 cents. |
| 46. " between 17 and 18 years. | 61. " nearly 11 per cent. |
| 47. " in a little over 14 years. | 62. " nearly 28 per cent. |









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